

Gravitational-wave progenitors

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Lecture 13

NCU, Summer Semester 2024





*Previously
on GW-progenitors...*

Population synthesis on *binaries*

- **NOT** the same thing as binary evolutionary simulations

meaning: 'detailed' evolutionary computations e.g. with MESA

(yes, MESA can run binaries too)

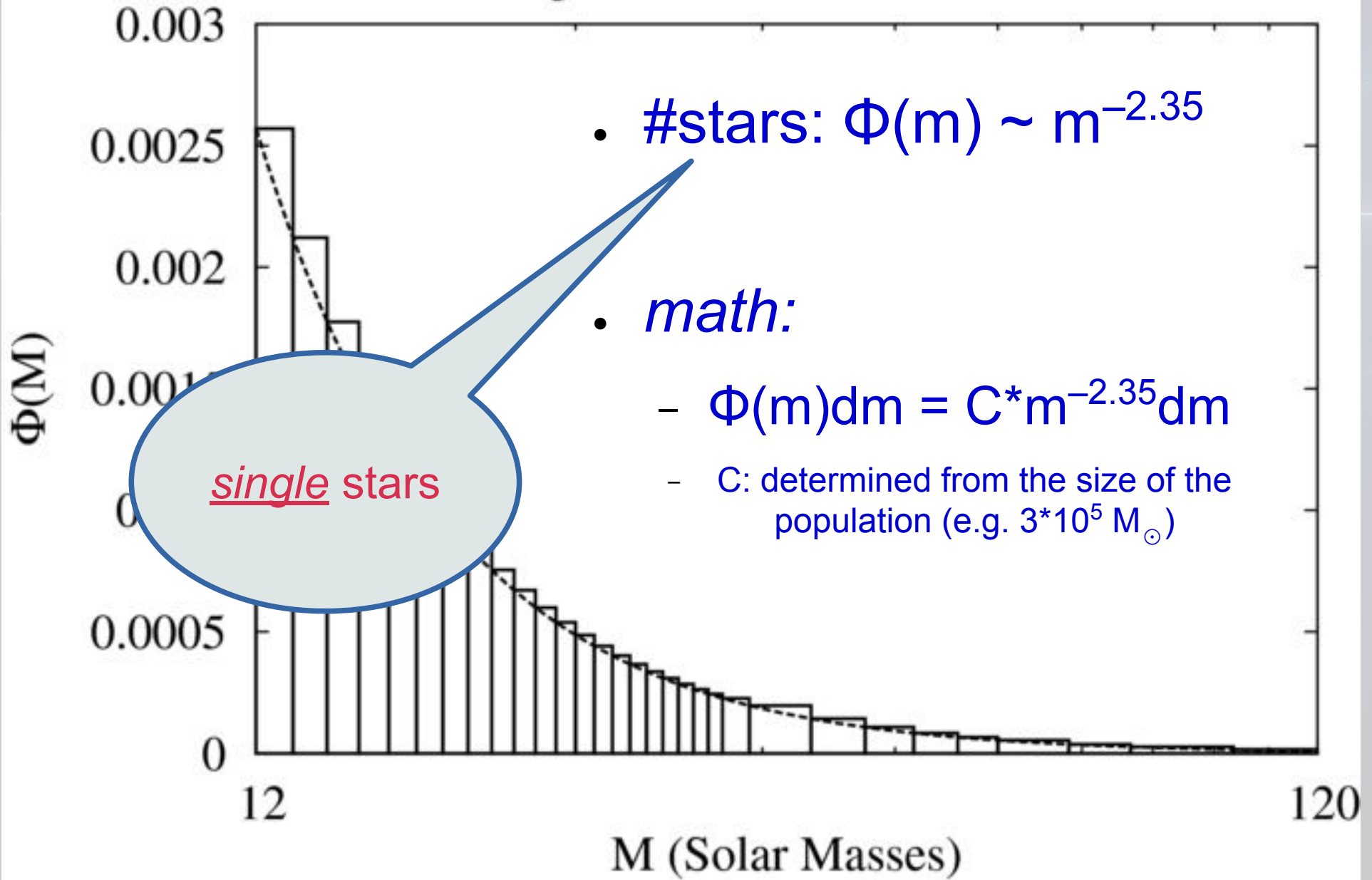
Remember the Initial Mass Function (IMF)?

Pop.synth. starts with that.

But binaries make life complicated.

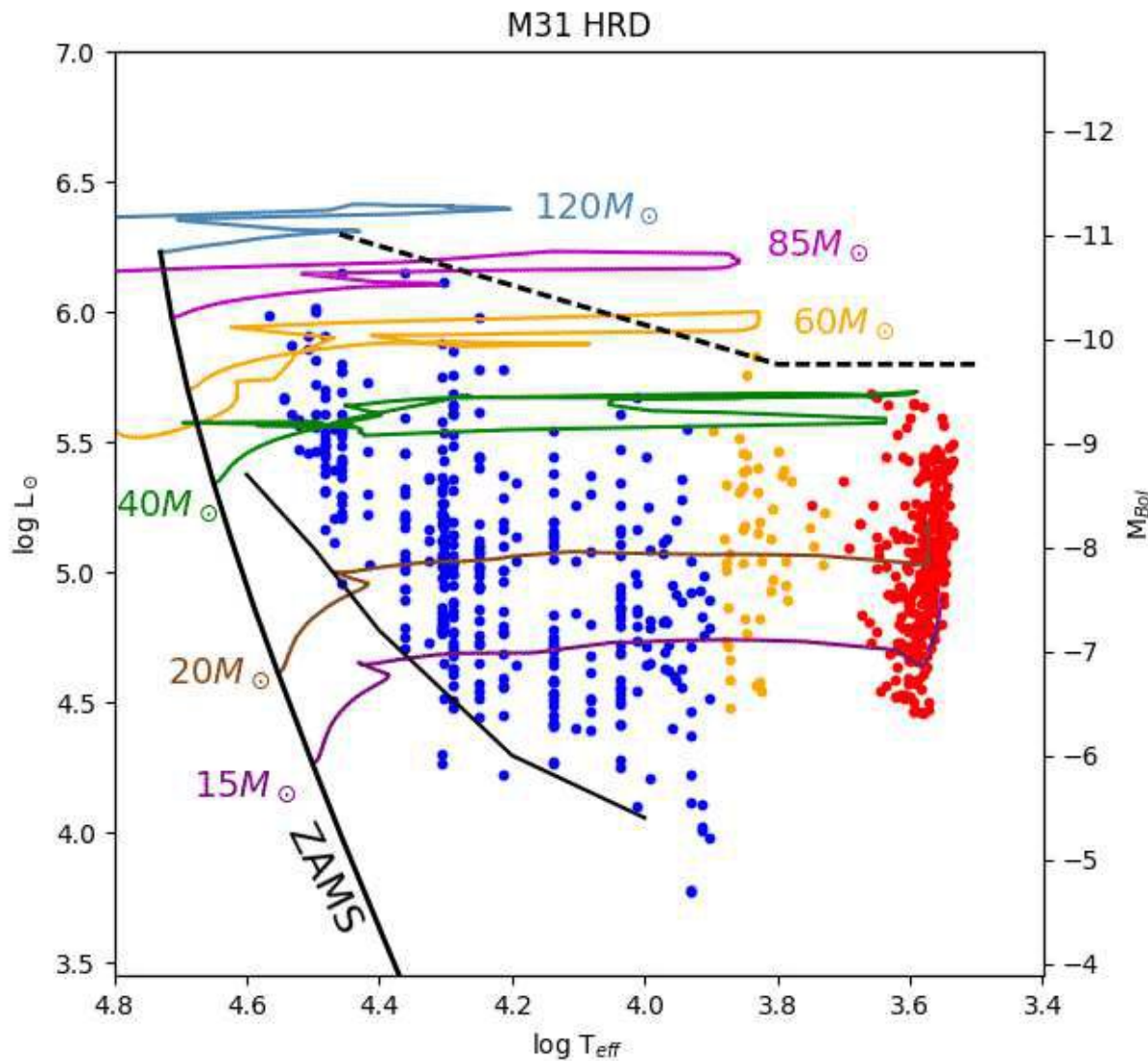
REMINDER: The Initial Mass Function (IMF)

Salpeter Initial Mass Function

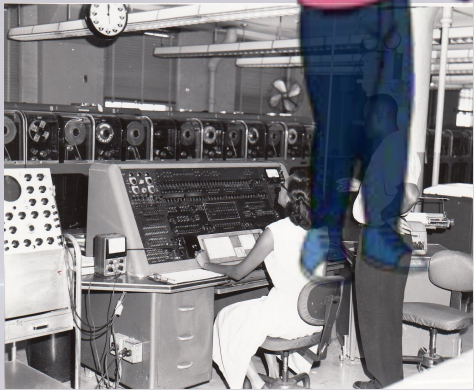


(*single*
stars)

Let's think!



- How would you “convert” between the lines and the dots?
- *Meaning:*
 - how would you compare theoretical predictions with observations?



HR-diagram

Age, Mass, Radius, T_{eff} [K], $\log(L/L_{\odot})$, Massloss rate...



```
1# MIST version number = 10.1
2# MESA revision number = 11701
```

```
3# -----
4# Yinit      Zinit  [Fe/H]  [a/Fe]  v/vcrit
5# 0.2511    1.42857E-03  -1.00   0.00   0.00
```

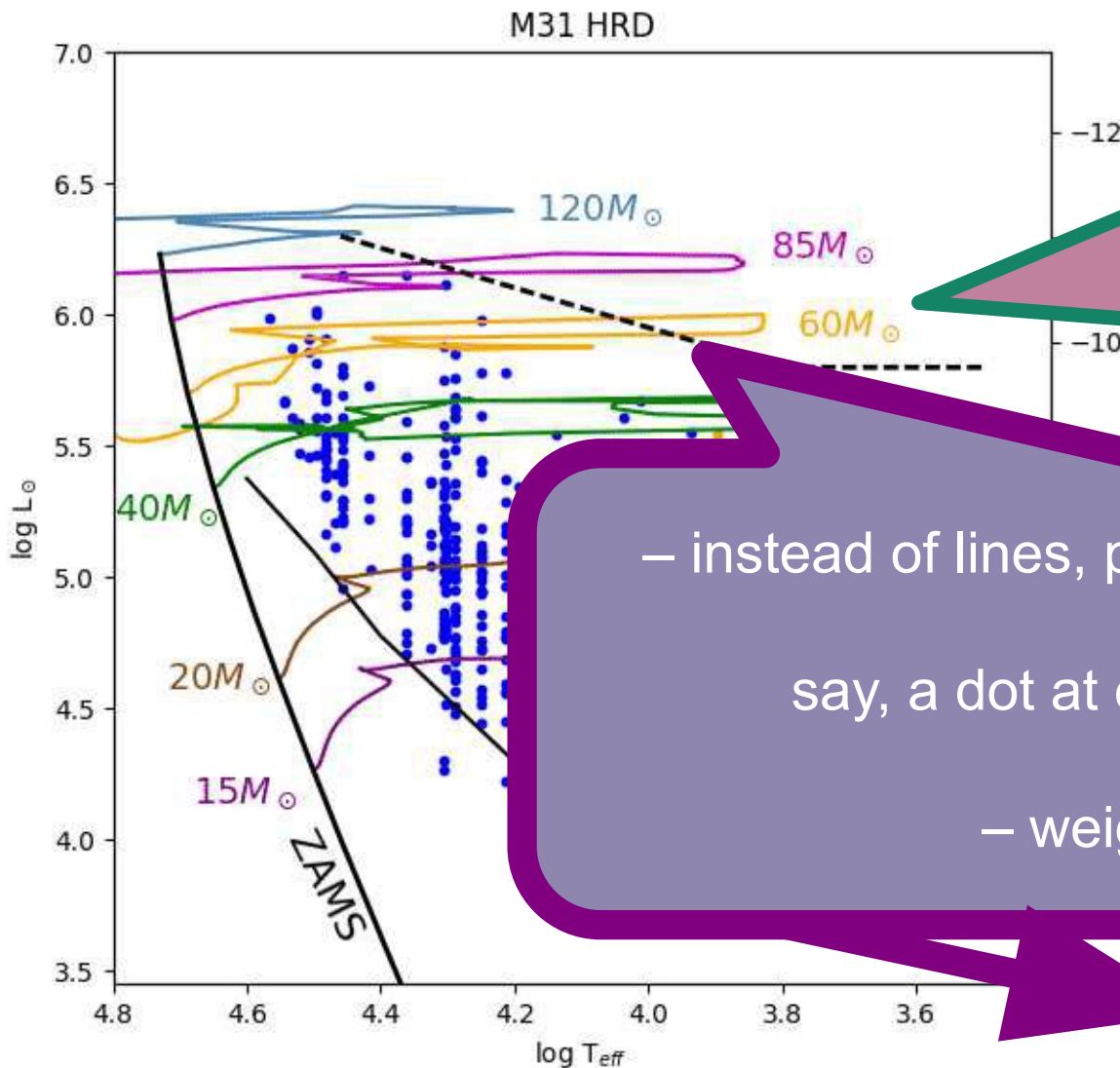
```
6# -----
7#      initial mass  N_pts  N_EEP  N_col  phase  type
8# 1.9999727046E+01    808      8    73    YES  high-mass
9# EEPs:      1    202    353    454    605    631    707    808
```

```
10# -----
11#                                     1                                     2                                     3                                     4                                     5
12#                                     star age                               star mass                               star mdot                               log dt                               he_core mass
13# 2.7320575584293762E+005          1.9999727045763130E+001          -6.6667141481350412E-009          4.6121780058570057E+000          0.0000000000000000E+000
14# 2.7345019073205121E+005          1.9999725407394834E+001          -6.6668930715861210E-009          4.6125719424045064E+000          0.0000000000000000E+000
15# 2.7369462562116480E+005          1.9999723769026541E+001          -6.6670719950372001E-009          4.6129658789520063E+000          0.0000000000000000E+000
16# 2.7393906051027833E+005          1.9999722130658245E+001          -6.6672509184882791E-009          4.6133598154995070E+000          0.0000000000000000E+000
17# 2.7418349539939192E+005          1.9999720492289949E+001          -6.6674298419393581E-009          4.6137537520470087E+000          0.0000000000000000E+000
18# 2.7442793028850551E+005          1.9999718853921653E+001          -6.6676087653904380E-009          4.6141476885945094E+000          0.0000000000000000E+000
19# 2.7467236517761904E+005          1.9999717215553360E+001          -6.6677876888415162E-009          4.6145416251420093E+000          0.0000000000000000E+000
20# 2.7491680006673269E+005          1.9999715577185061E+001          -6.6679666122925961E-009          4.6149355616895100E+000          0.0000000000000000E+000
21# 2.7516123495584622E+005          1.9999713938816765E+001          -6.6681455357436759E-009          4.6153294982370108E+000          0.0000000000000000E+000
22# 2.7540566984495980E+005          1.9999712300448472E+001          -6.6683244591947550E-009          4.6157234347845106E+000          0.0000000000000000E+000
23# 2.7565010473407339E+005          1.9999710662080176E+001          -6.6685033826458340E-009          4.6161173713320123E+000          0.0000000000000000E+000
24# 2.7589453962318692E+005          1.9999709023711880E+001          -6.6686823060969130E-009          4.6165113078795130E+000          0.0000000000000000E+000
25# 2.7613897451230051E+005          1.9999707385343584E+001          -6.6688612295479929E-009          4.6169052444270129E+000          0.0000000000000000E+000
26# 2.7638340940141404E+005          1.9999705746975291E+001          -6.6690401529990719E-009          4.6172991809745136E+000          0.0000000000000000E+000
27# 2.7662784429052763E+005          1.9999704108606995E+001          -6.6692190764501510E-009          4.6176931175220144E+000          0.0000000000000000E+000
28# 2.7687227917964122E+005          1.9999702470238695E+001          -6.6693979999012308E-009          4.6180870540695151E+000          0.0000000000000000E+000
29# 2.7711671406875481E+005          1.9999700831870403E+001          -6.6695769233523099E-009          4.6184809906170159E+000          0.0000000000000000E+000
30# 2.7736114895786840E+005          1.9999699193502106E+001          -6.6697558468033889E-009          4.6188749271645166E+000          0.0000000000000000E+000
31# 2.7760558384698193E+005          1.9999697555133814E+001          -6.6699347702544679E-009          4.6192688637120174E+000          0.0000000000000000E+000
32# 2.7785001873609552E+005          1.9999695916765514E+001          -6.6701136937055478E-009          4.6196628002595173E+000          0.0000000000000000E+000
```


(*single*
stars)

Let's think!

number ratio of
MS vs. RSG
stars



- pick lines according to IMF
(cf. initial mass column)
- compute how much time they all spend as blue stars
- and how much as red stars

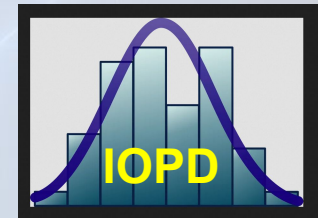
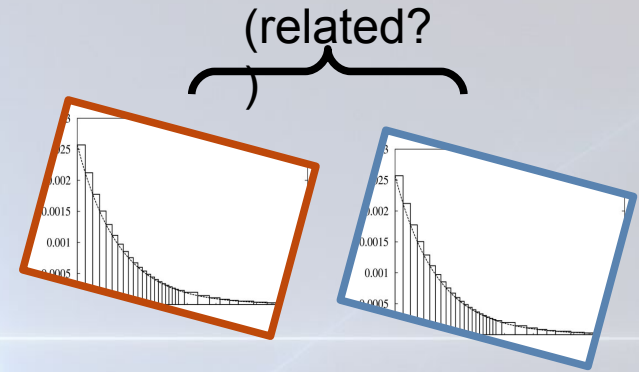
- instead of lines, plot the evolutionary tracks as dots!
say, a dot at every 10 thousand year
- weight with the IMF

an actual (simulated)
stellar population!

simulated =
"synthetic"

Population synthesis on *binaries*

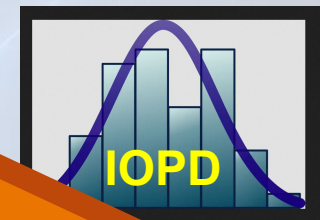
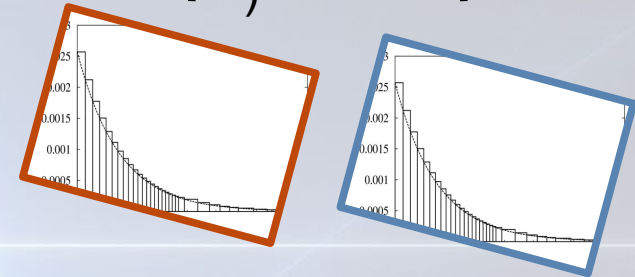
- 2 stars instead of 1
 - both have their individual IMFs
- orbital separation!
 - Initial Orbital Period Distribution
same kind of thing as the IMF but for the period,
i.e. an observation-based statistical distribution
- plus a *lot* of assumptions about the evolution
 - mass transfer (stable/unstable? conservative/non-conservative? ...)
 - Common Envelope phase (outcome: merger or survival?
separation afterwards?)
 - supernova physics... and the kick.



Population synthesis on *binaries*

- 2 stars instead of 1
 - both have their own individual IMFs
- orbital separation
 - Initial Orbital Period
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- plus a *lot* of assumptions about the evolution
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 - Common Envelope phase (outcome: merger or separation afterwards?)
 - supernova physics... and the kick.

(related?)



under active research

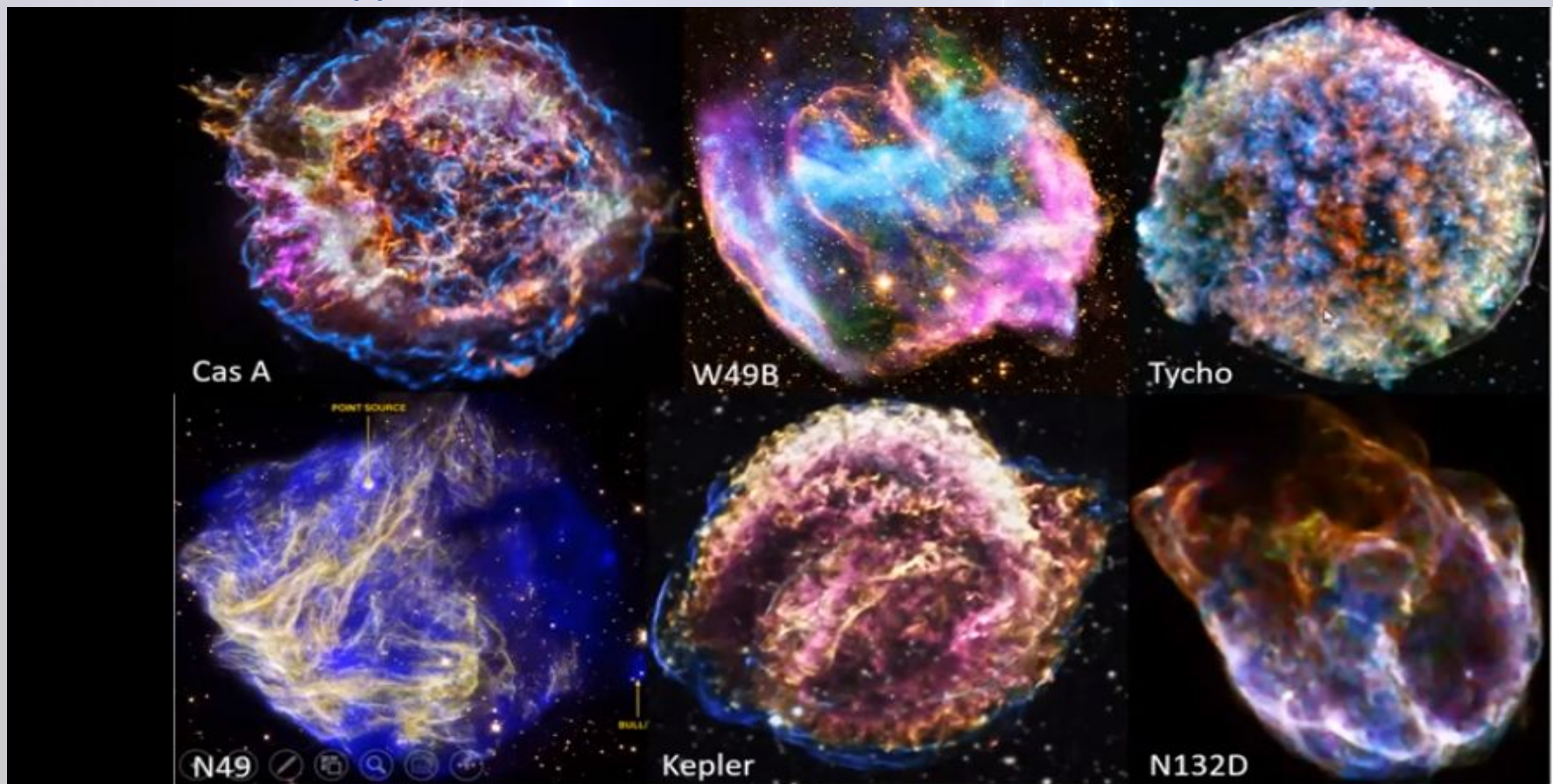
on top of what
we
already don't
know
about *single*
stars'
evolution

Kicks

- happens for single-star supernovae too
= natal kick
which happen when the NS is born

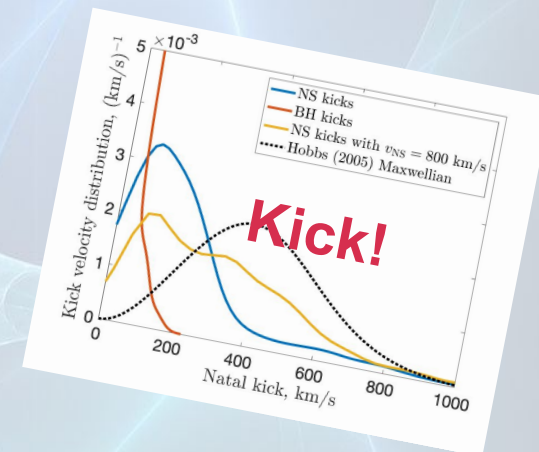
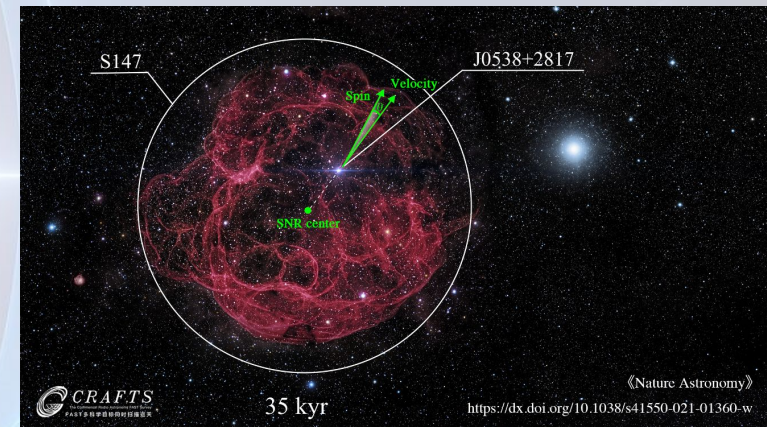


Each color corresponds to different emission process.



Kicks

- happens for single-star supernovae too
= natal kick
which happen when the NS is born
also see: *pulsar kick*, *NS kick*, *SN kick*
 - needs: assymetric explosion
- in binaries, one SN may kick out the companion
- survival rate is uncertain
 - but in pop.synth., drawn from a – *you guessed it* – statistical distribution :D

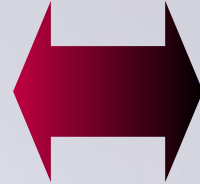


cf. Mandel & Müller (2020)

IMPORTANT . . .

Exam
warning!
:P

- Stellar evolution modelling



- Synthetic population modelling

- based on first principles
(*5 stellar equations*)
- follows one star's life at the time
- IMF is not yet considered
- result is *a line* ('track') in the HR-diagram

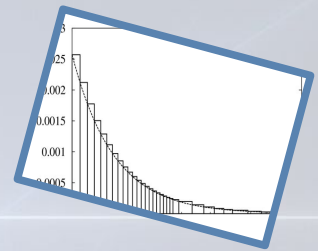
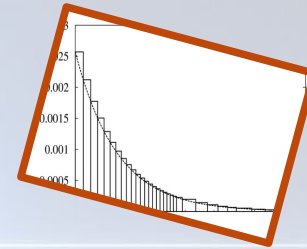
- relies on stellar evolution modelling
- does not simulate the individual star's life (typically)
- IMF is taken into account
- result is a *statistically meaningful* prediction about a *population*



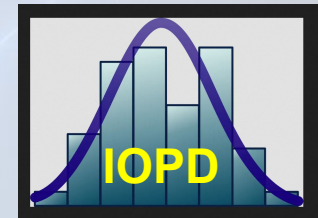
Today...

...the last steps!

Star-formation history

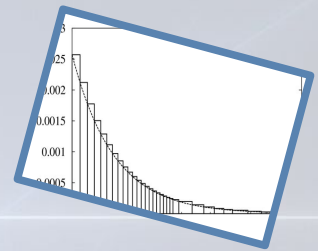
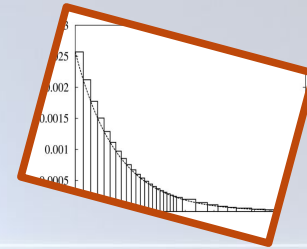


Not enough!

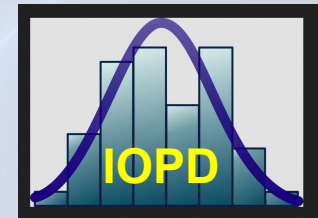


Star-formation history

- We need to know the *history* of how the stars are being born....

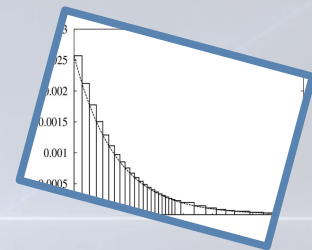
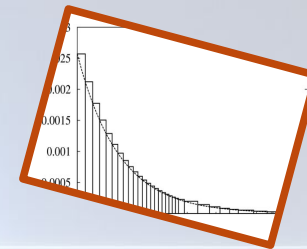
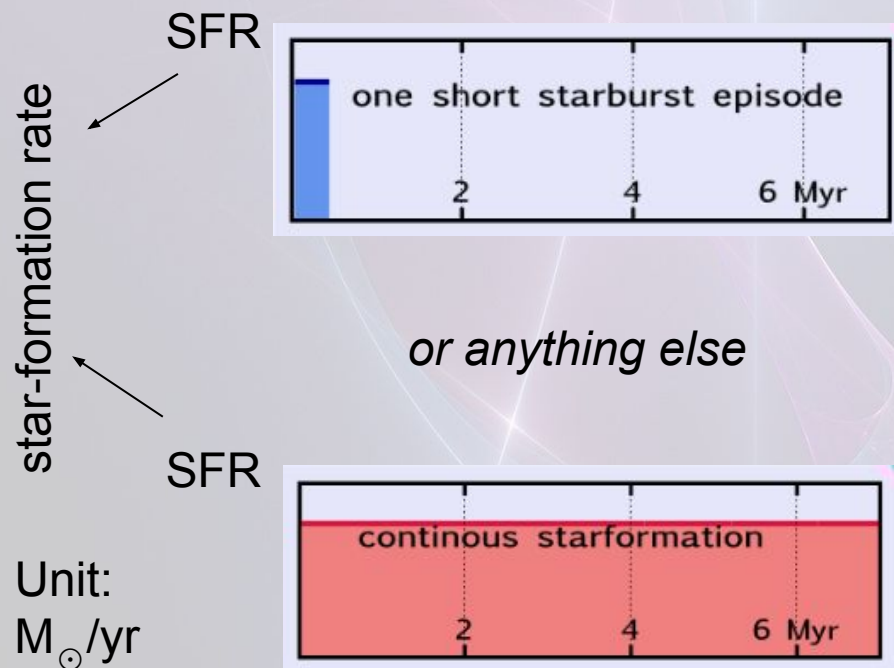


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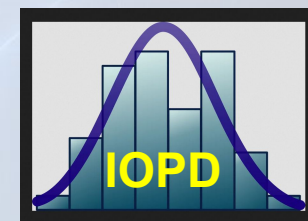


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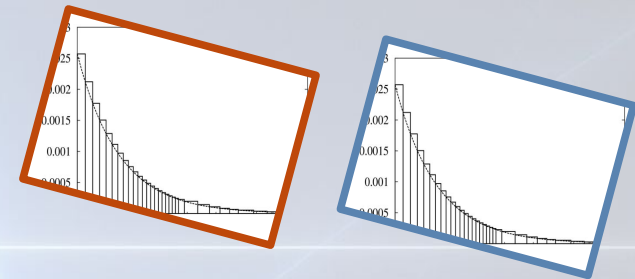
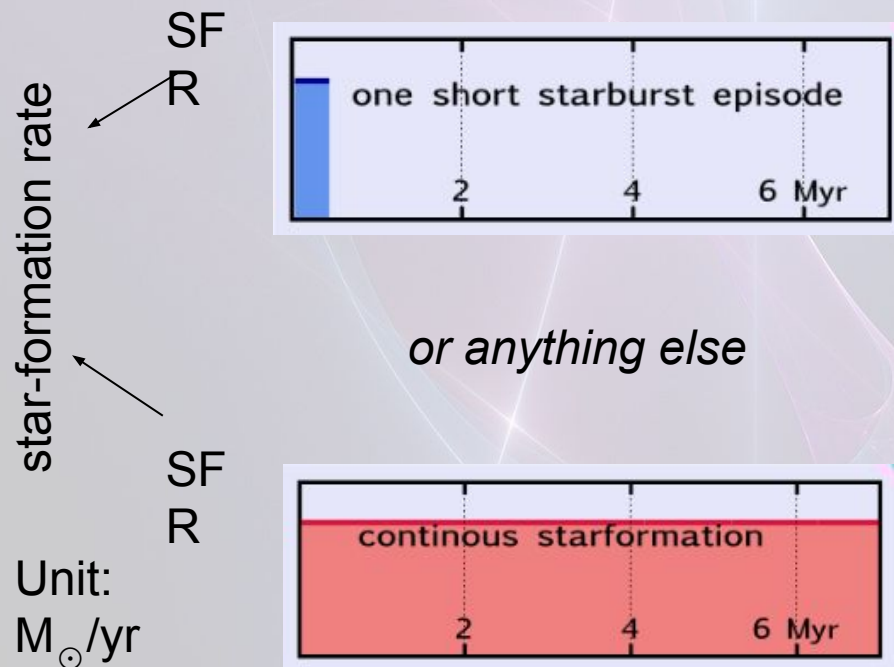


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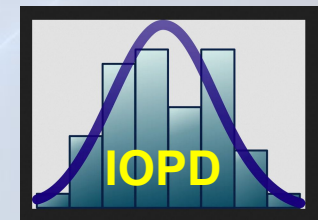


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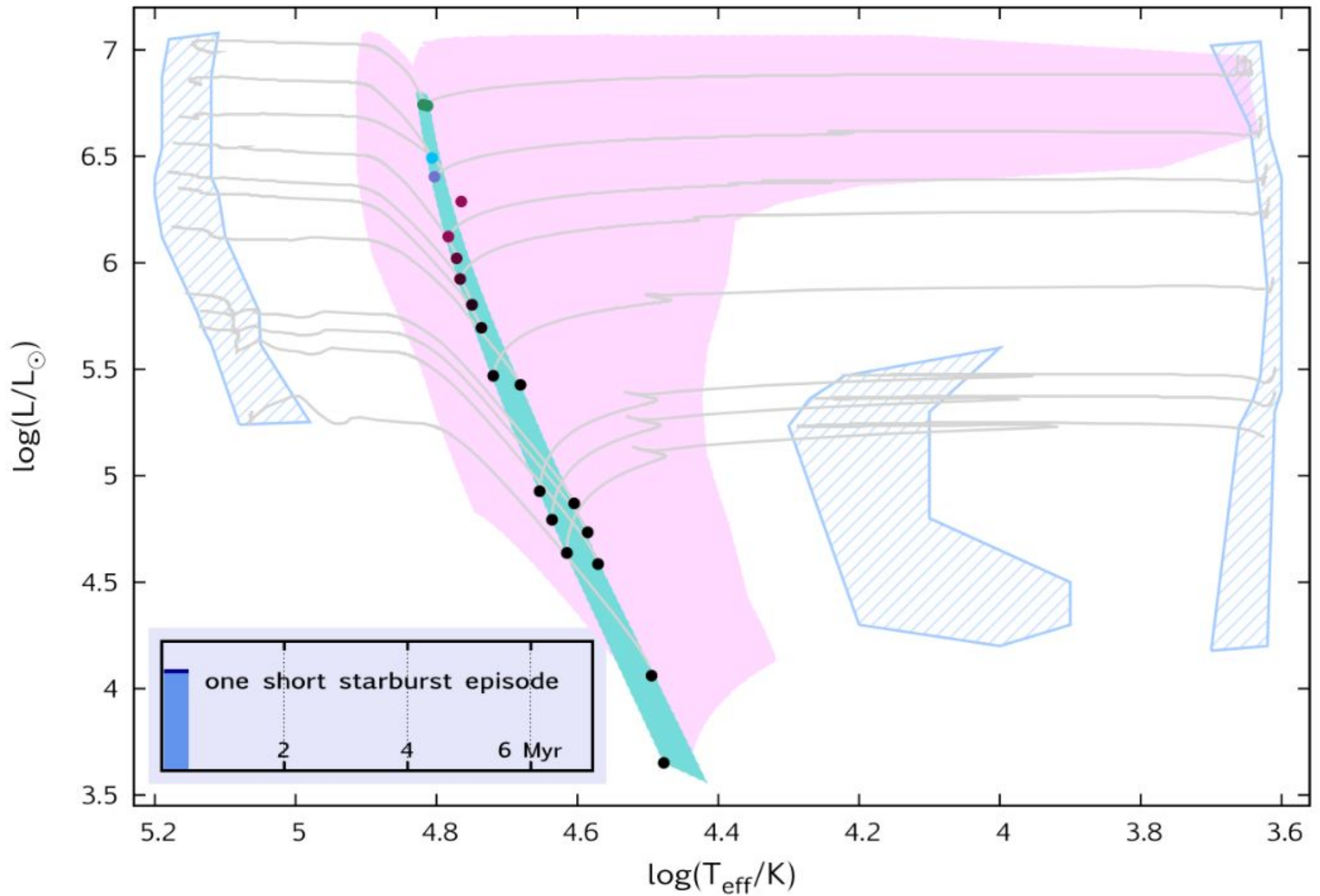


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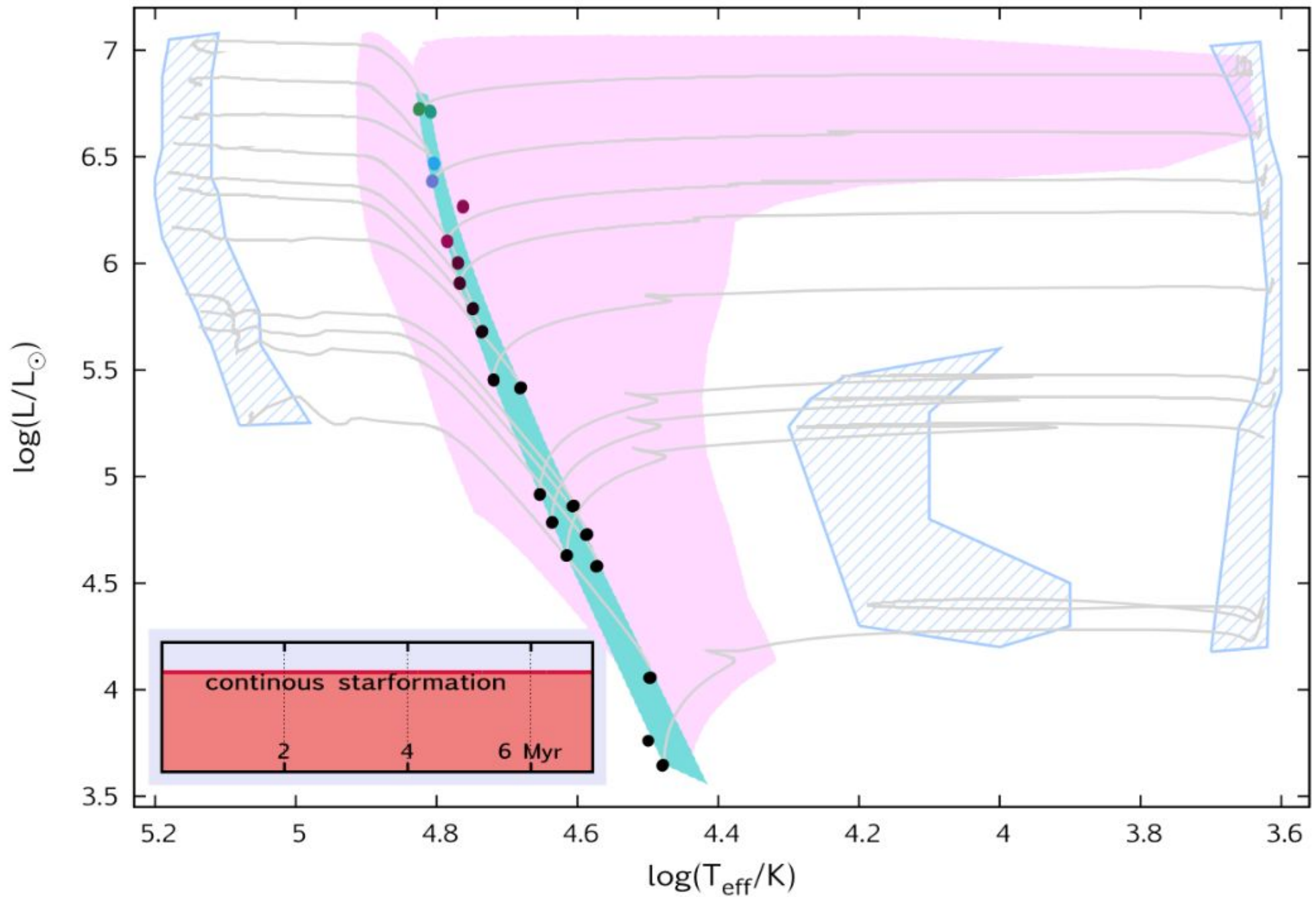


We need all these to do
(binary) population synthesis.

Simulating a galaxy... or starcluster

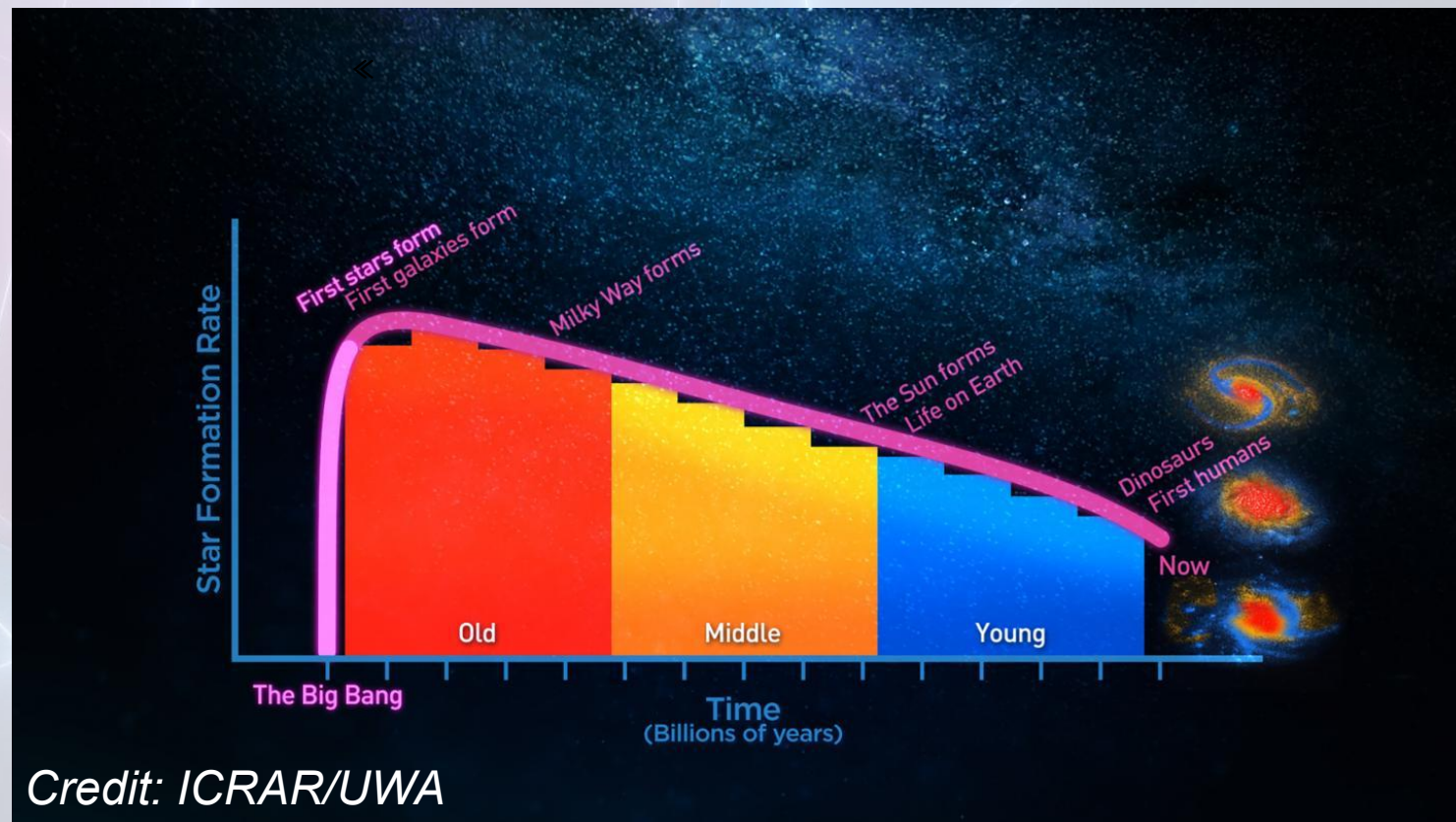


Simulating a galaxy... or starcluster



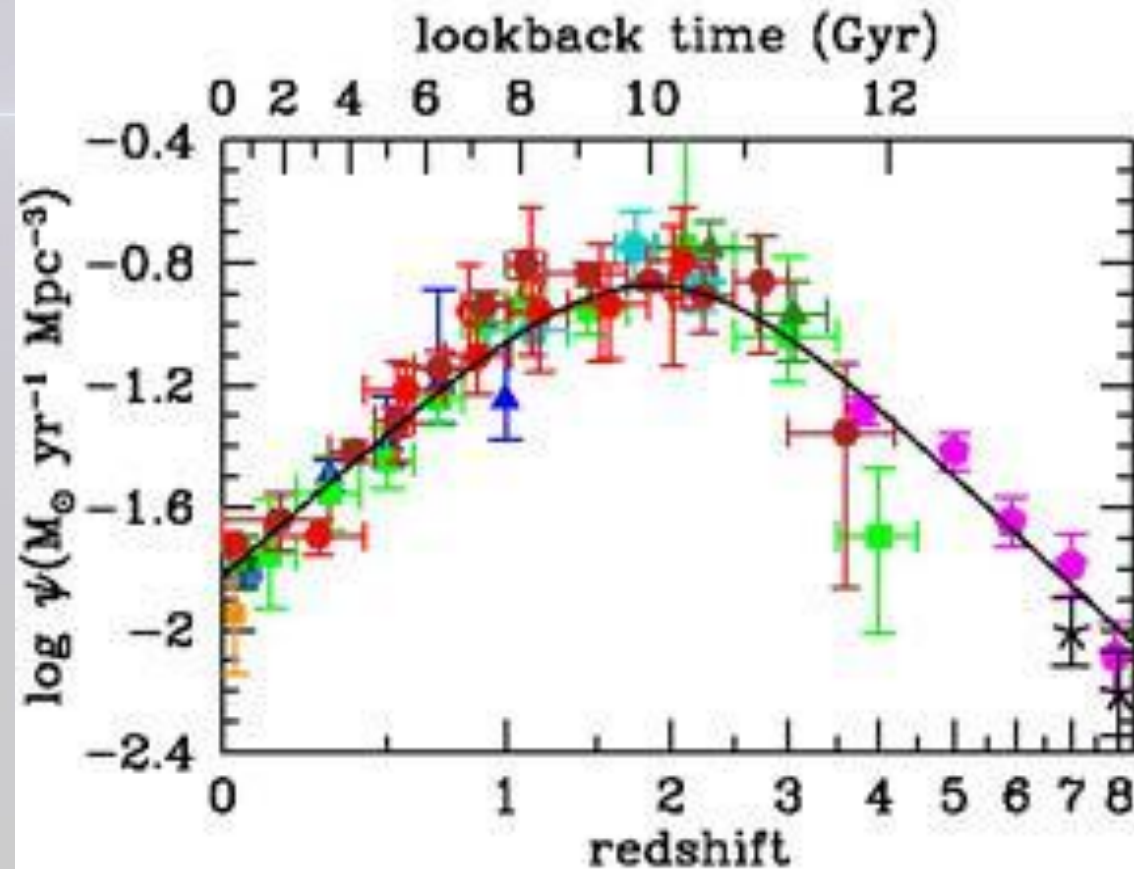
From star-formation history to *cosmic* star-formation history

- This is what we need to predict GW-event rates from synthetic populations

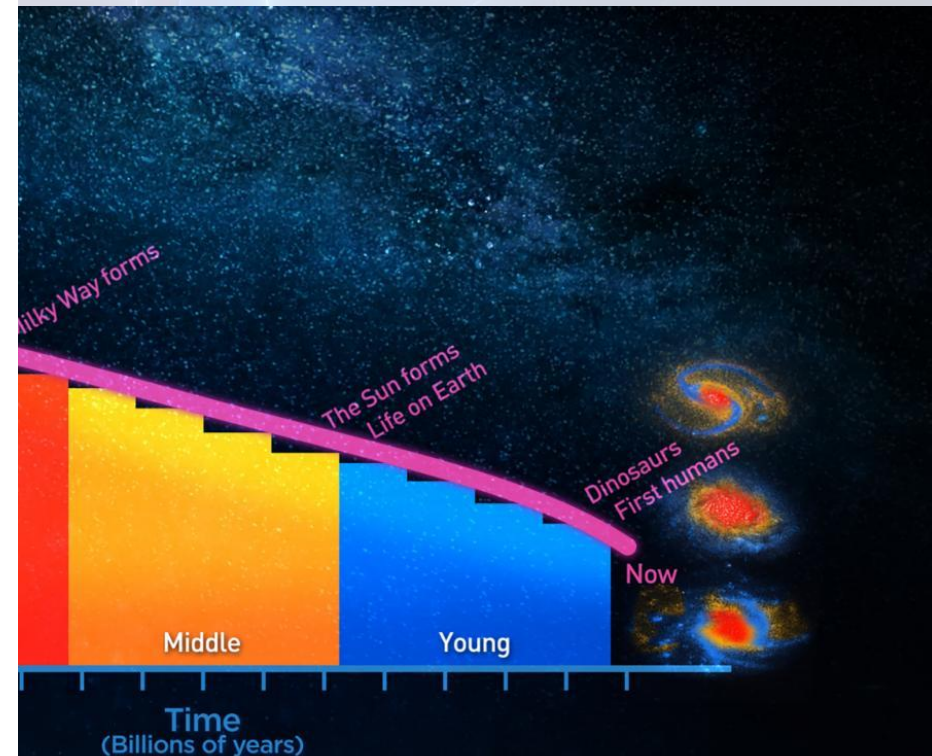


From star-formation history to *cosmic* star-formation history

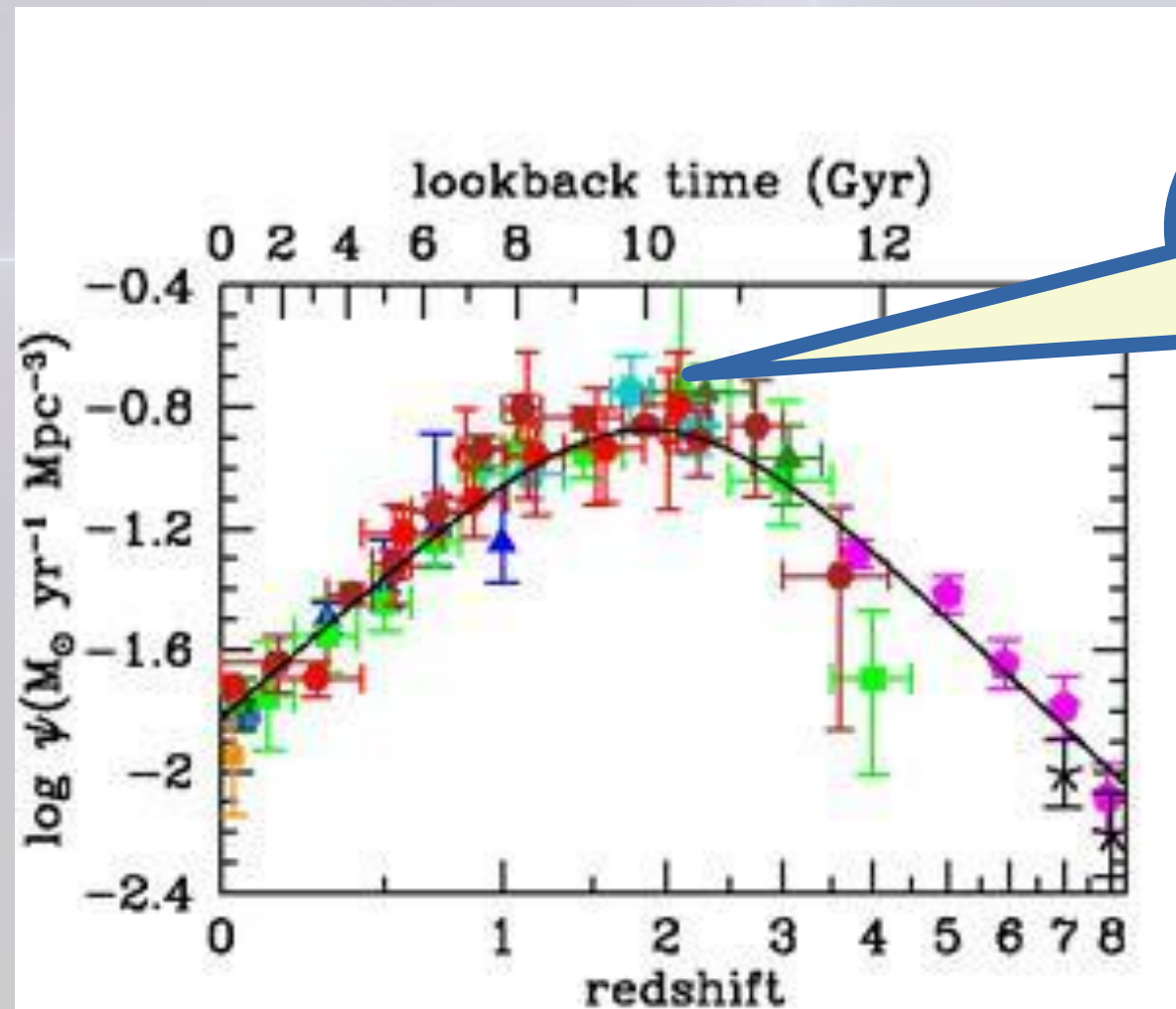
predict GW-event rates



Credit: Madau & Dickinson (2014)

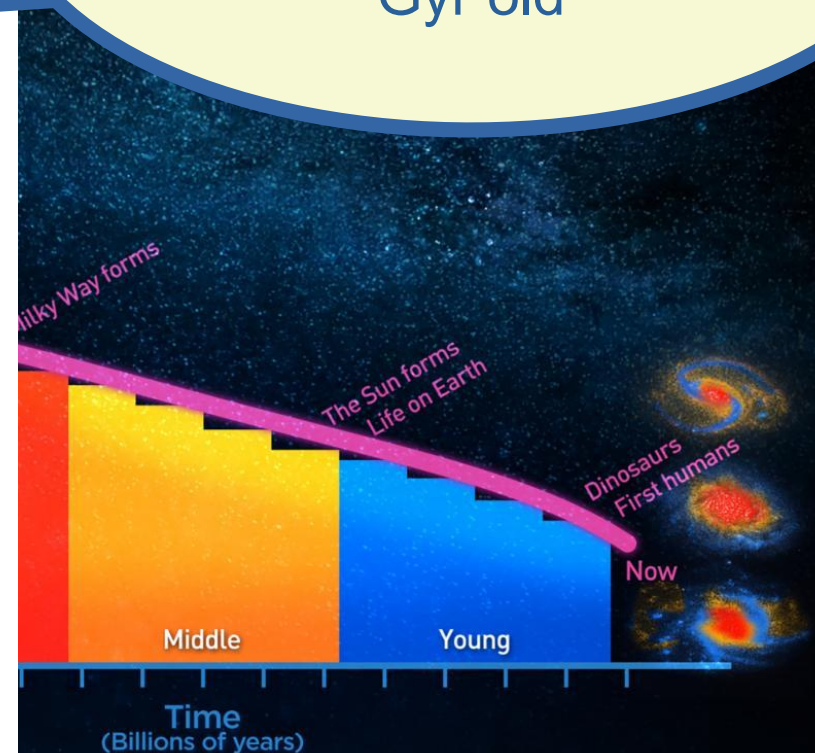


From star-formation history to *cosmic* star-formation history

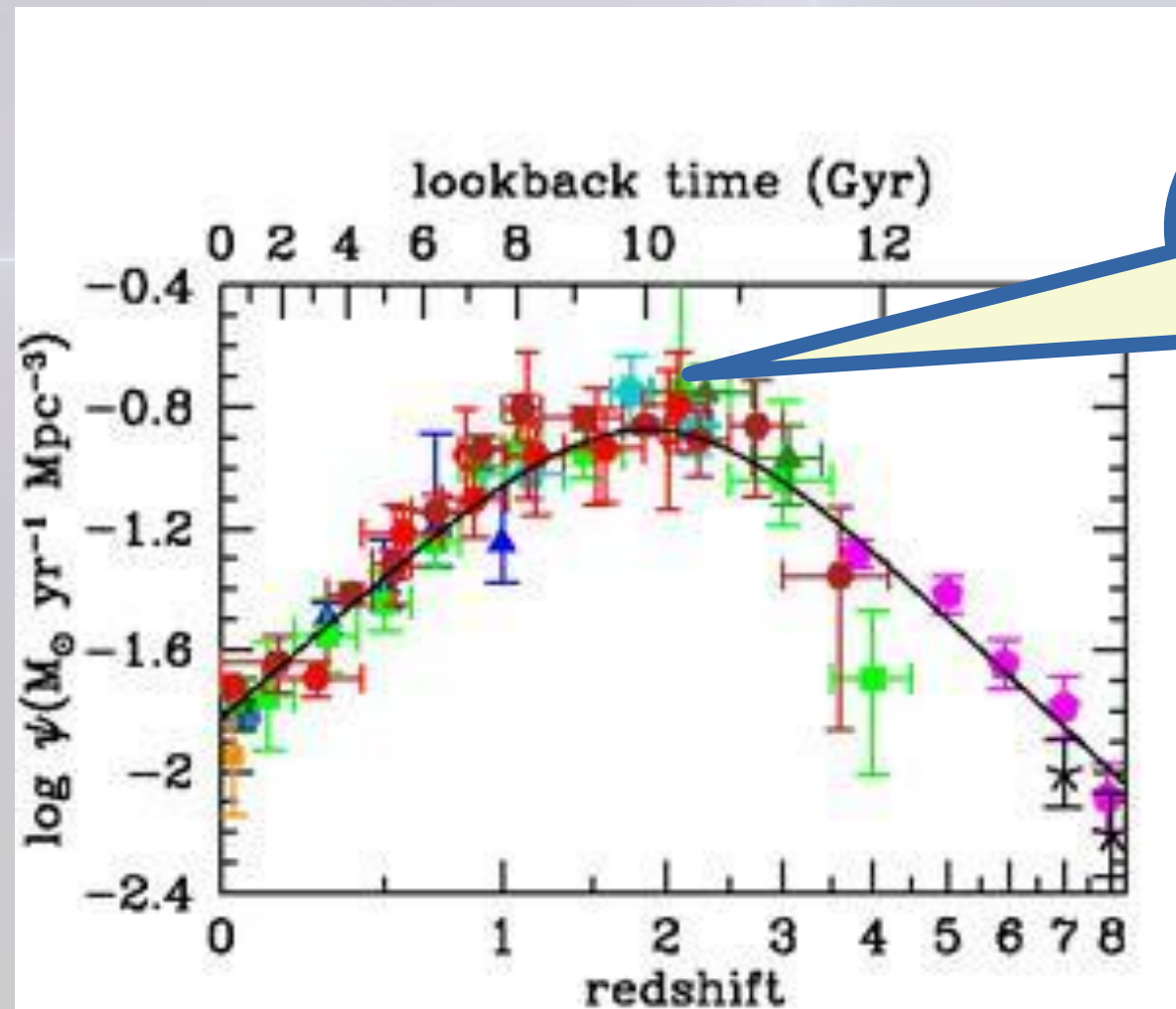


Peaking: somewhere around $z = 2$, when the Universe was ~ 3.5 Gyr old

Credit: Madau & Dickinson (2014)



From star-formation history to *cosmic* star-formation history



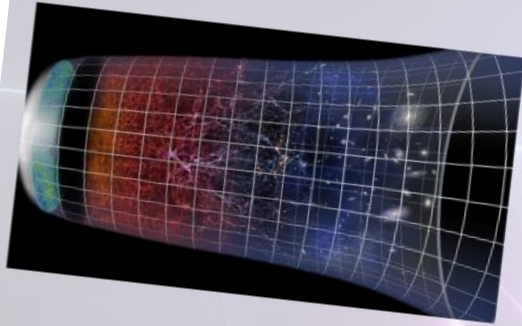
Peaking: somewhere around $z = 2$, when the Universe was ~ 3.5 Gyr old

Note:
massive stars live short lives!
 $2\text{-}20 \text{ Myr} \ll 13 \text{ Gyr}$

Credit: Madau & Dickinson (2014)

Now we can answer the original ^(kind of)
question of this whole lecture series

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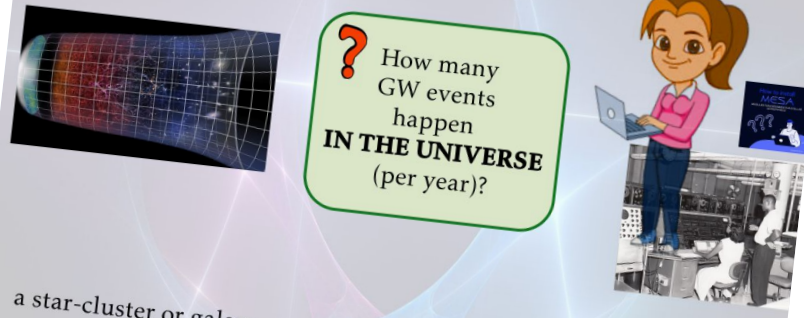


? How many
GW events
happen
IN THE UNIVERSE
(per year)?



a star-cluster or galaxy: one star-formation event of size (e.g.) $10^7 M_{\odot}$
aLIGO/Virgo detectors observe GWs from the whole Universe...

Now we can answer the original ^(kind of) question of this whole lecture series



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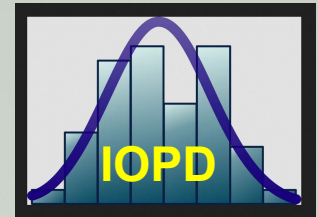
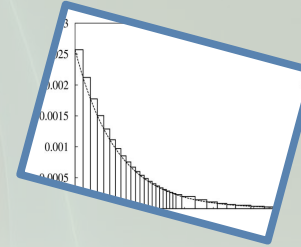
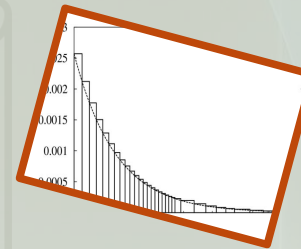
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stellar models

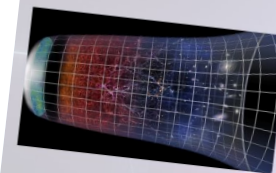
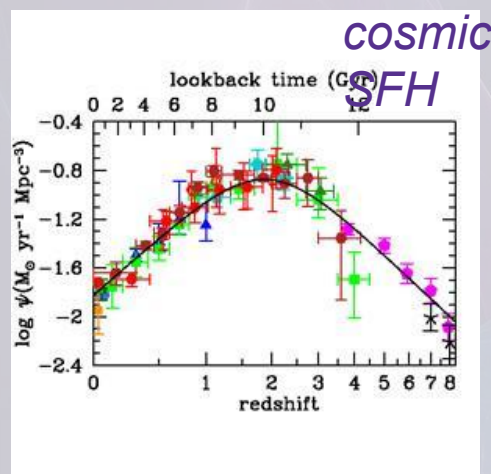
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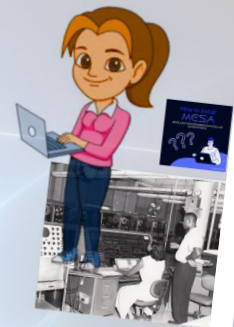



initial distributions

+ a lot of assumptions about binary physics



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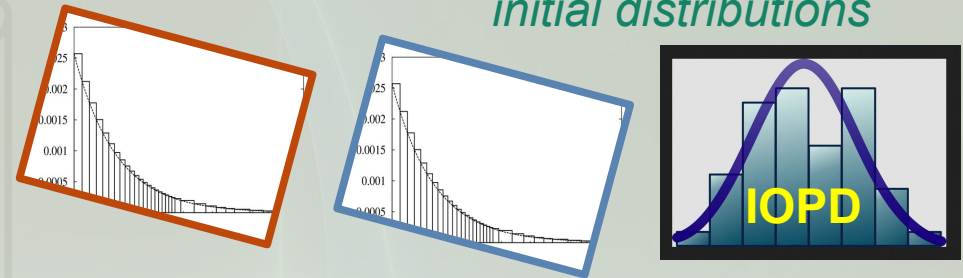
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stellar models

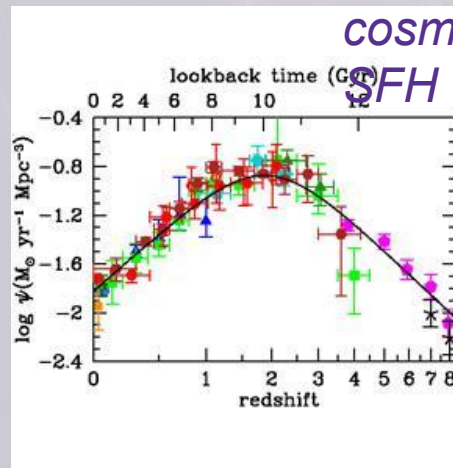


initial distributions



+ a lot of assumptions about binary physics

cosmic SFH



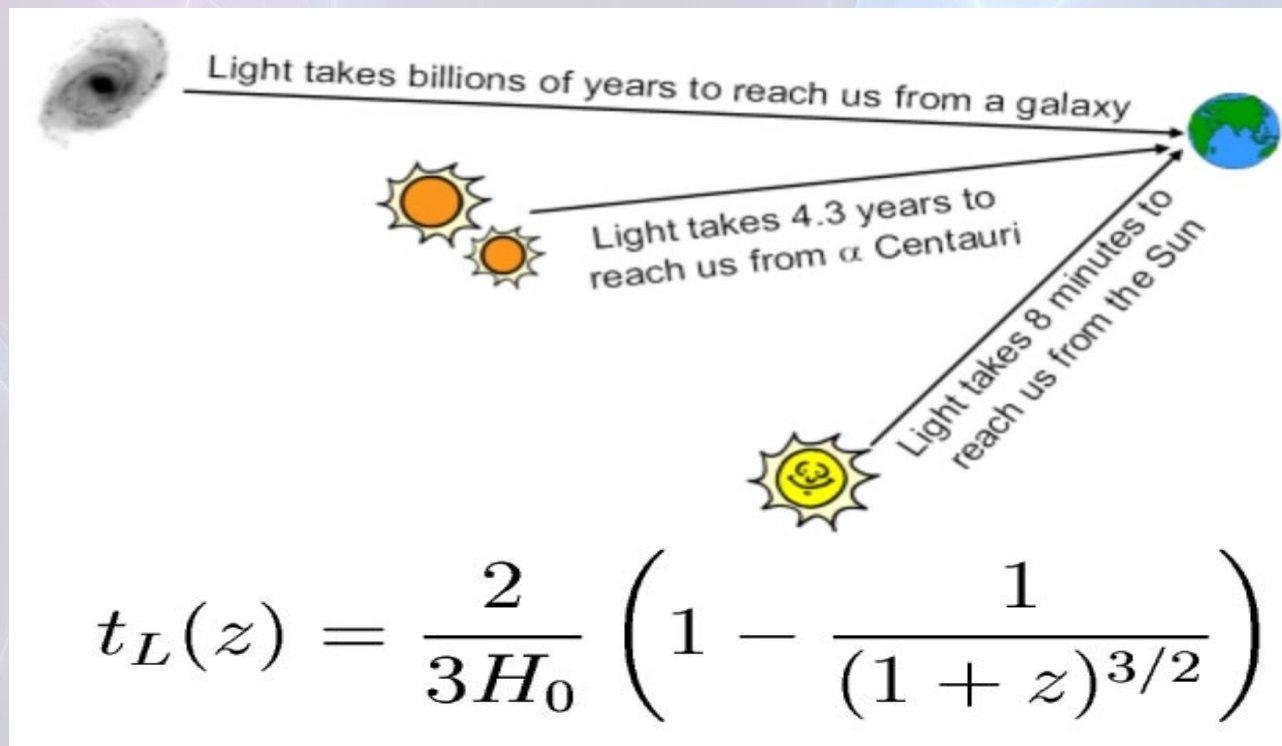
Important piece of math:
Convolution
of two functions

$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau.$$

Some more terms

Lookback time:

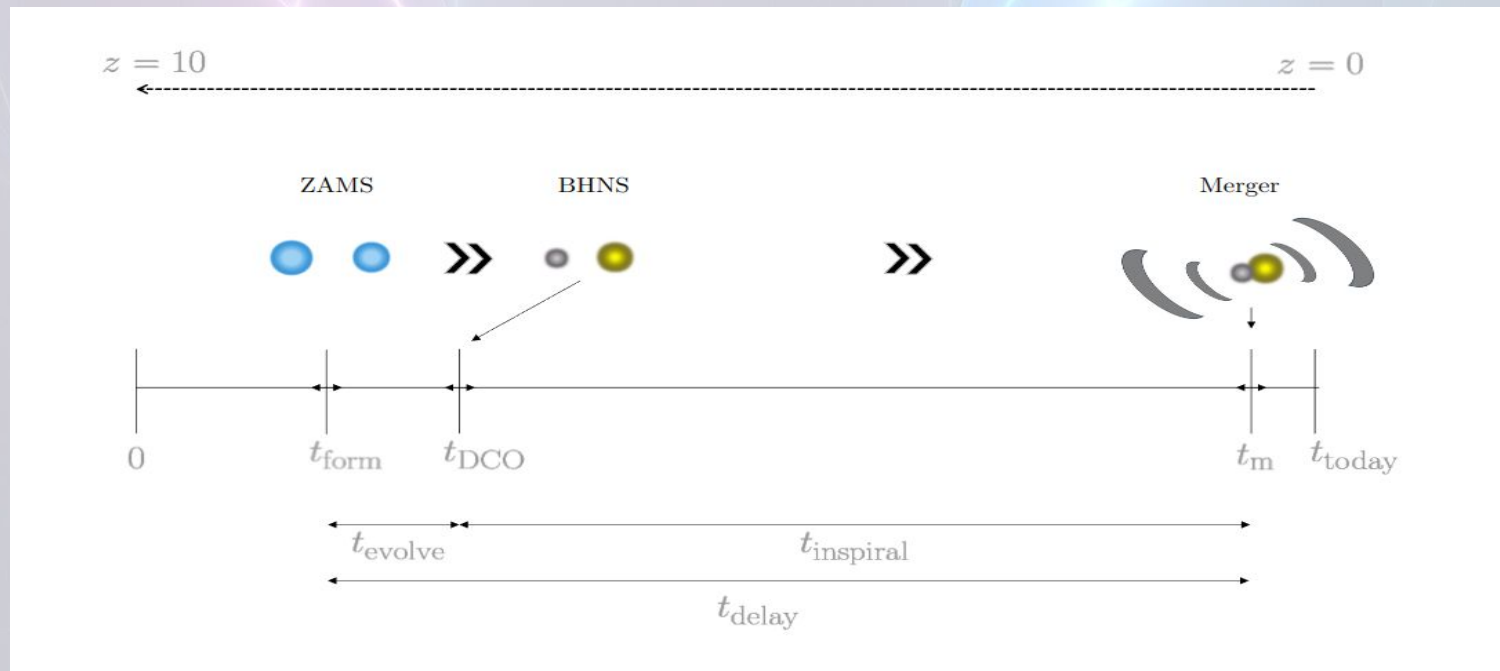
- difference between the age of the Universe *now* (at observation) and the age of the Universe when the photons* were emitted (from the given object).
*grav.waves



Some more terms

Delay time:

- the time it takes for a binary system to (1) evolve both stars, then (2) spiral in due to the emission of (undetectably weak) gravitational waves, and then (3) merge (emitting ((potentially)) detectable grav.waves). Typically: ~ 10 Myr – 13.77 Gyr



Even some more terms

- **Chirp mass**

$$\mathcal{M}_c \equiv \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

this is what can be directly derived from a measured GW-signal to derive m_1 and m_2 , a strong signal with good resolution is needed

- **Effective spin**

$$\chi_{\text{eff}} = \frac{M_1 \mathbf{a}_1 + M_2 \mathbf{a}_2}{M_1 + M_2} \hat{\mathbf{L}}$$

- **Orbital frequency**

$$f_{\text{orb}} = \frac{1}{2\pi} \sqrt{\frac{G(m_1 + m_2)}{a^3}}$$

- **Merger time**

$$t_{\text{merge}} = \frac{12}{19} \frac{c_0^4}{\beta} \int_0^{e_0} \frac{[1 + (121/304)e^2]^{\frac{1181}{2299}}}{e^{-29/19} (1 - e^2)^{3/2}} de$$

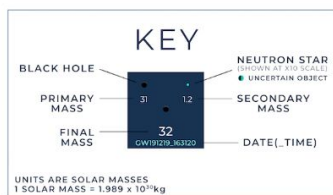
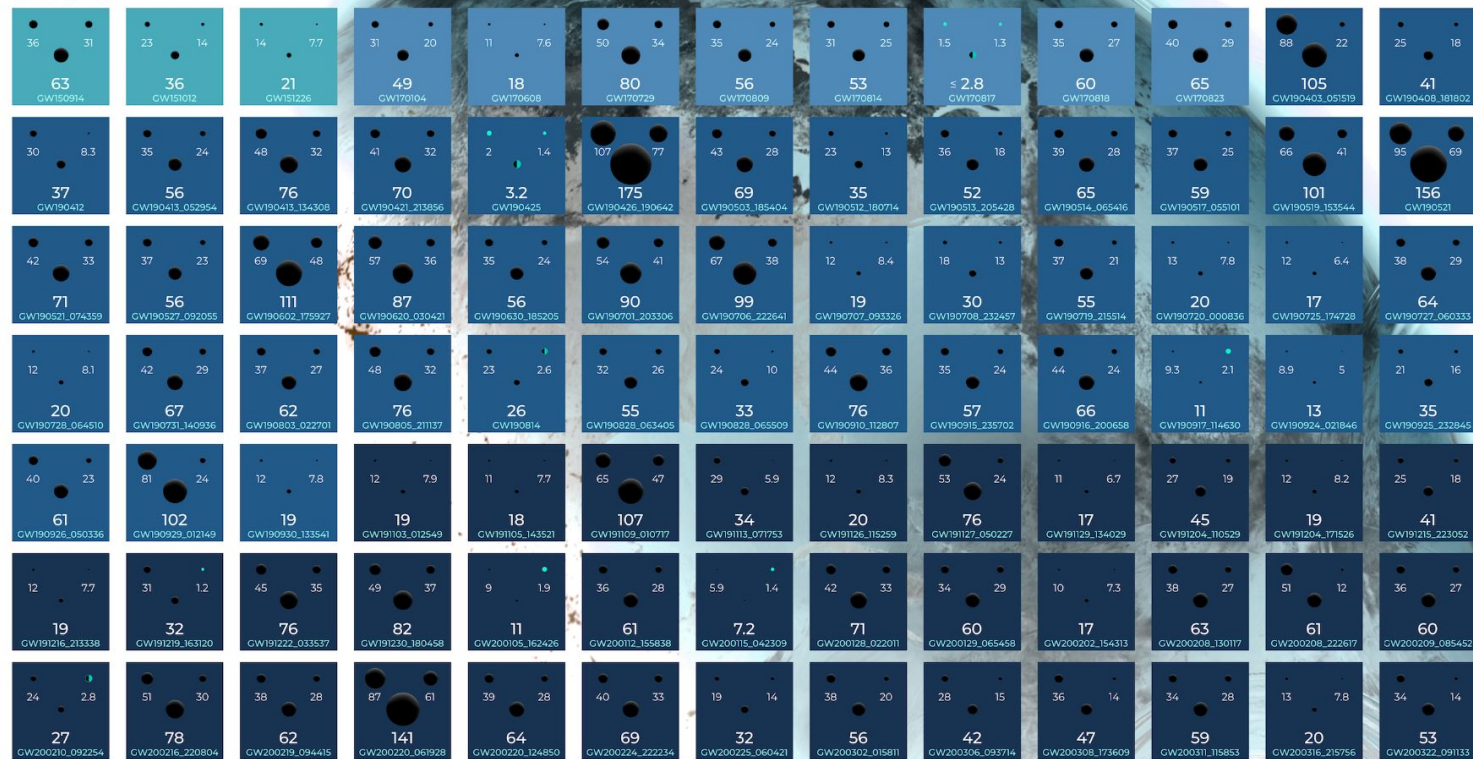
- **Delay time . . .**

The Gravitational-Wave Transient Catalogue 3 (GWTC-3)

**OBSERVING
01**
RUN
2015 - 2016

02
2016 - 2017

03a+b
2019 - 2020

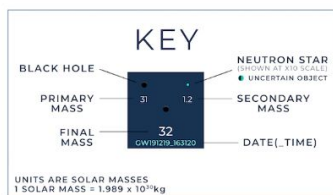
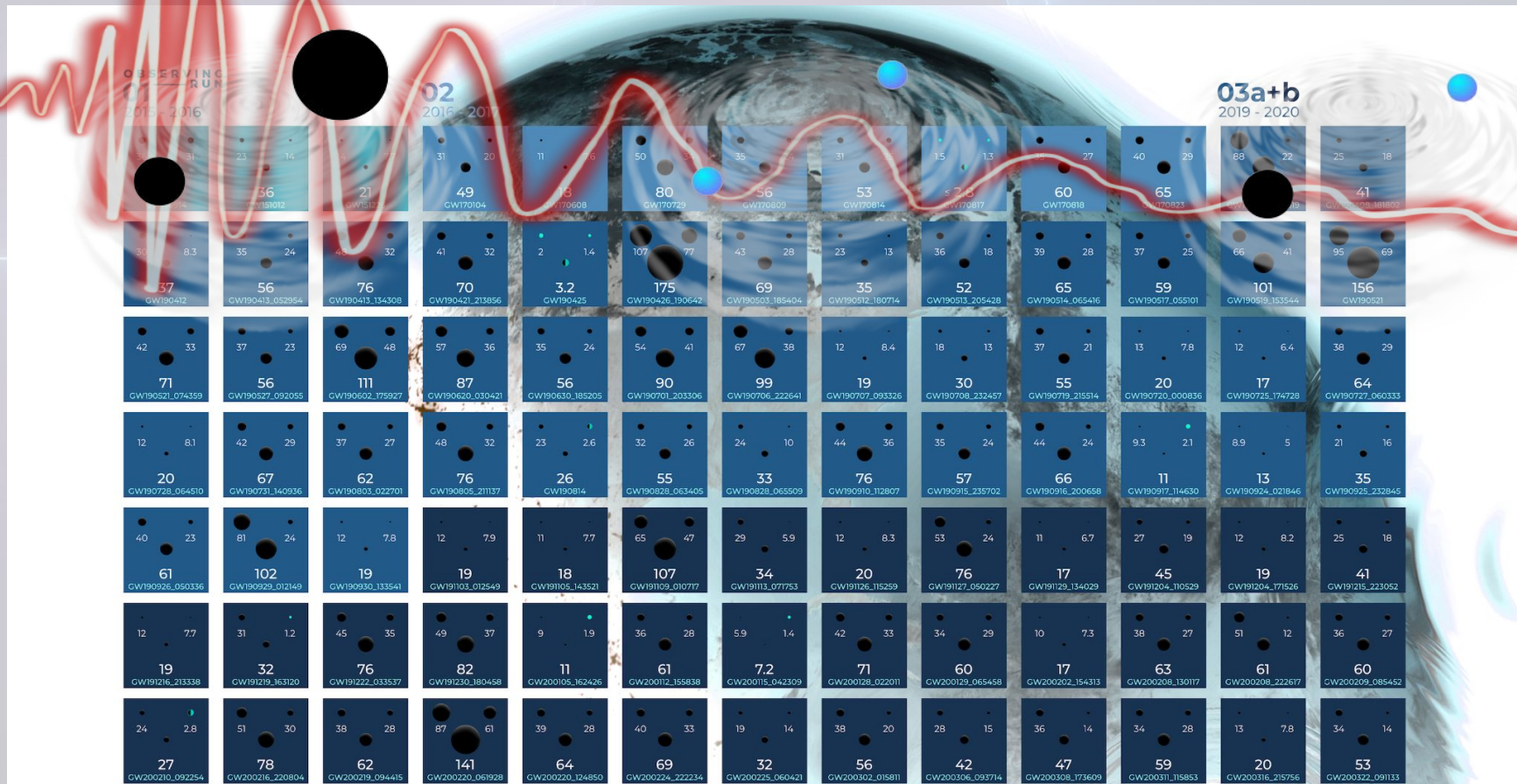


GRAVITATIONAL WAVE
MERGER
DETECTIONS
— SINCE 2015 —

AMC Center of Excellence for Gravitational Wave Discovery



The Gravitational-Wave Transient Catalogue 3 (GWTC-3)



Note that the mass estimates shown here do not include uncertainties, which is why the final mass is sometimes larger than the sum of the primary and secondary masses. In actuality, the final mass is smaller than the sum of the primary and secondary masses.

The events listed here pass one of two thresholds for detection. They either have a probability of being astrophysical of at least 90%, or they pass a false alarm rate threshold of less than 1 per 3 years.

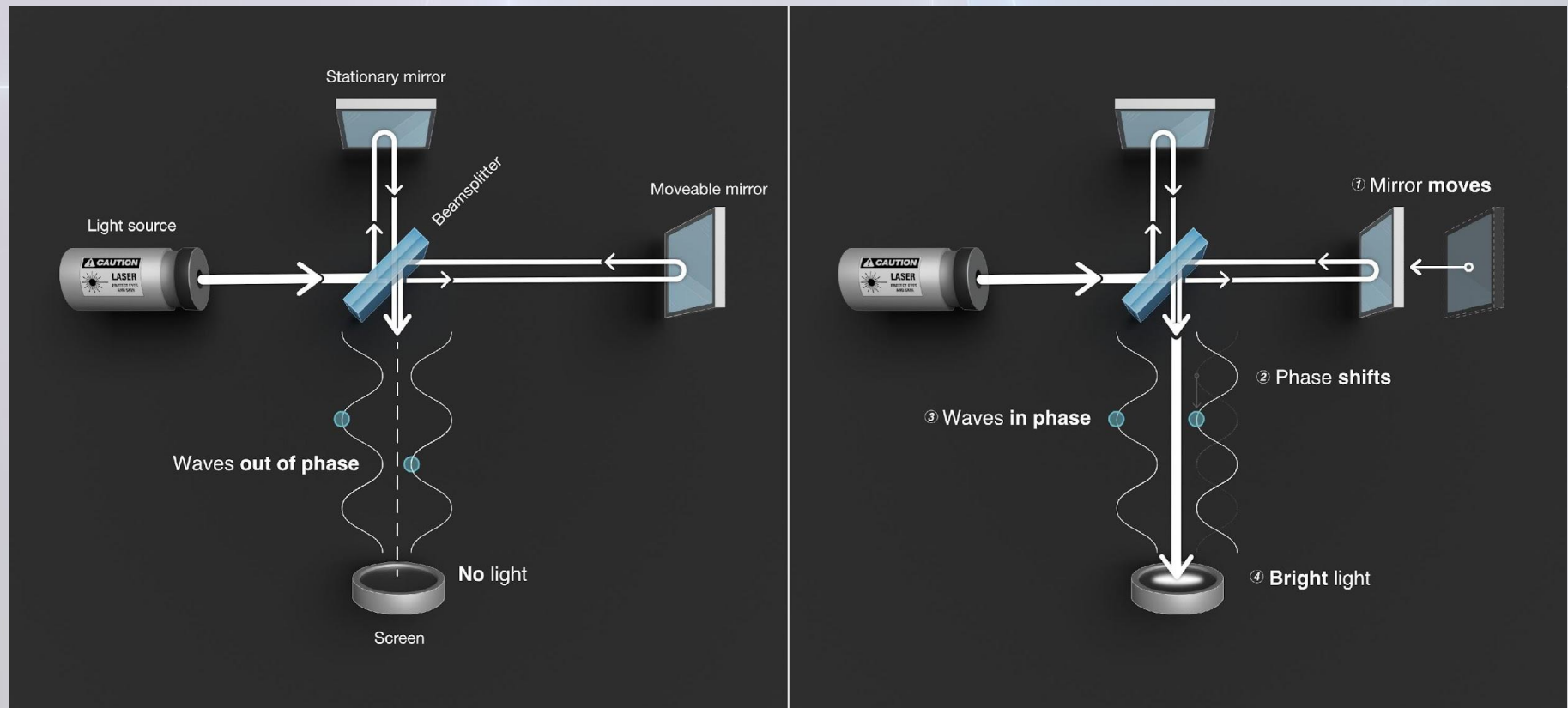
GRAVITATIONAL WAVE
MERGER
 DETECTIONS
 SINCE 2015

AMC Center of Excellence for Gravitational Wave Discovery



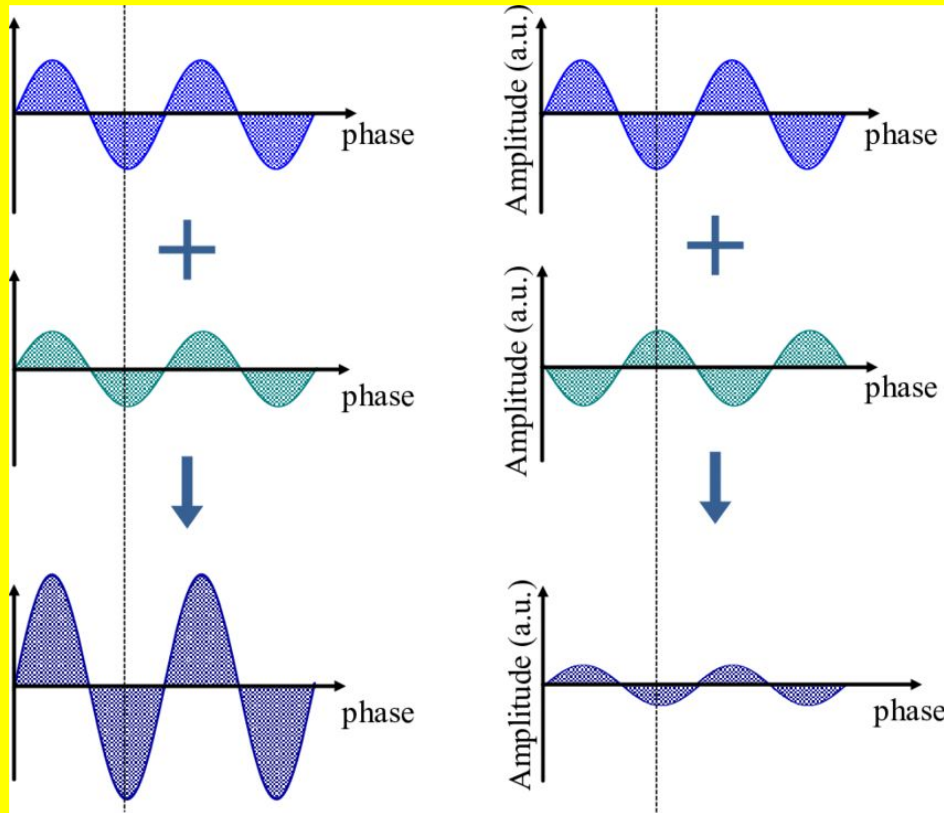
Detector

Interferometer:



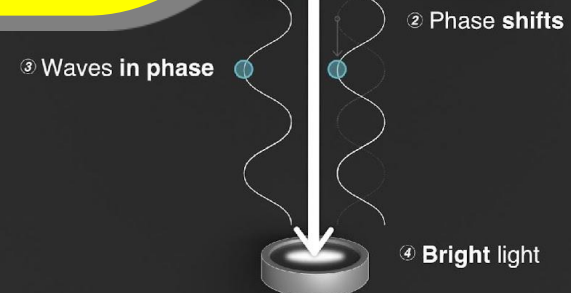
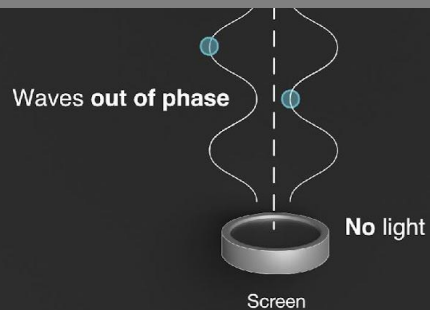
Interference

(of the *light* from the laser, not the GW signal!!)



can be directly
measured GW-signal
and m_2 , a strong
resolution is needed

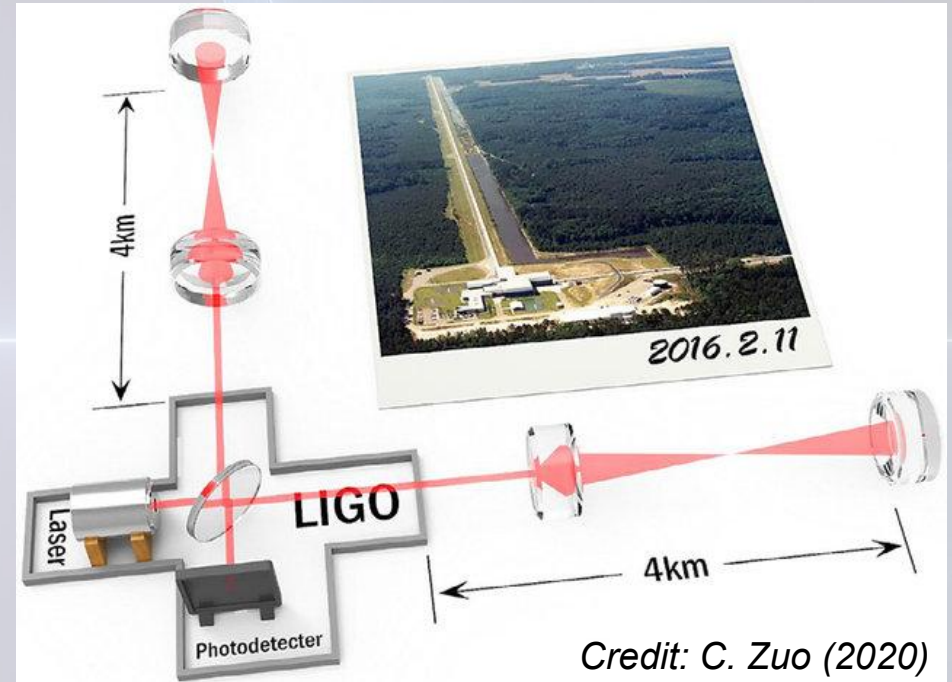
GW!



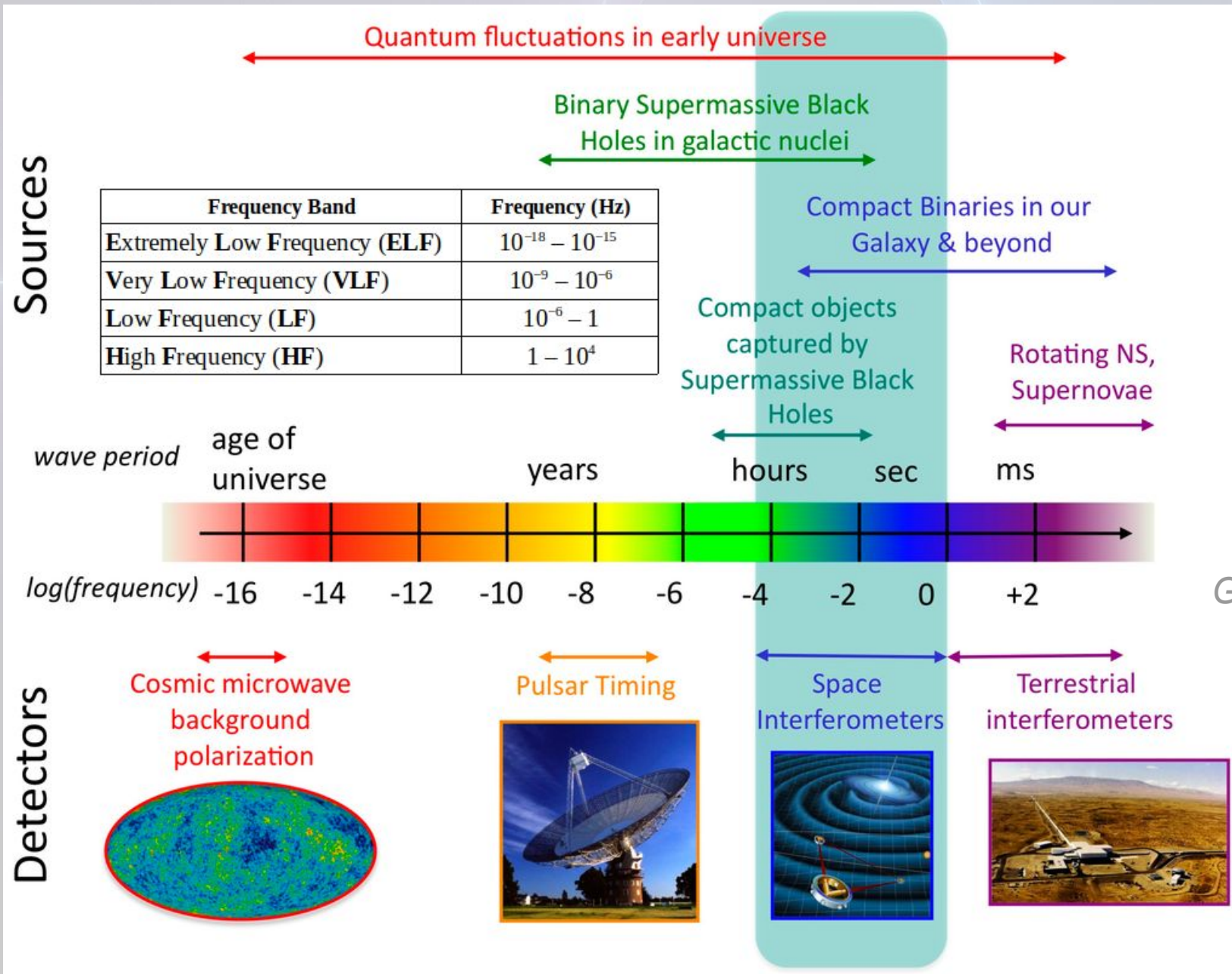
Credit: S. Kelley/NIST

And some names you MUST know

- LIGO:
 - Laser Interferometer Gravitational-wave Observatory (USA)
- aLIGO
 - advanced LIGO
 - the current version
- Virgo
 - LIGO's important little sister in Europe



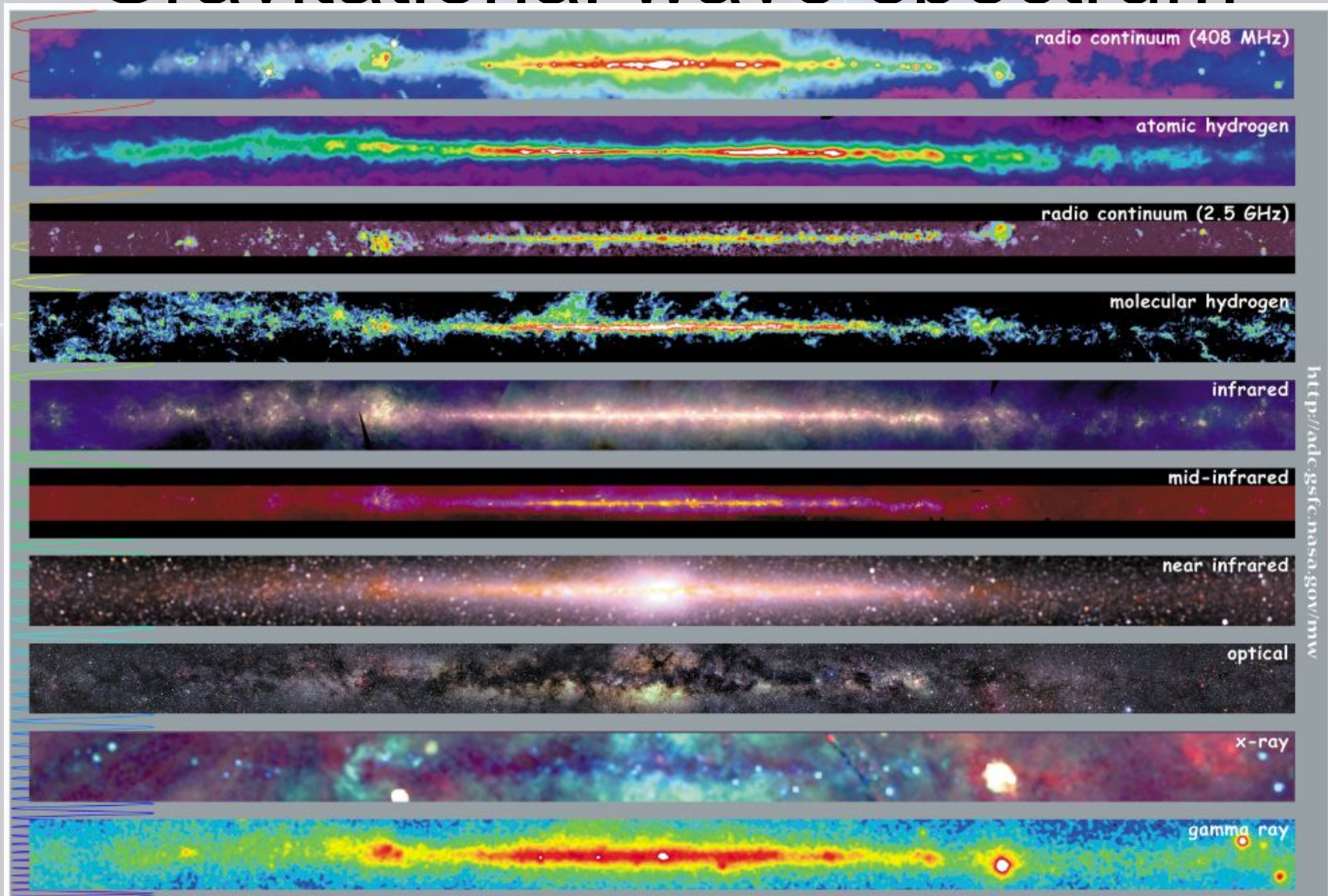
Gravitational wave spectrum



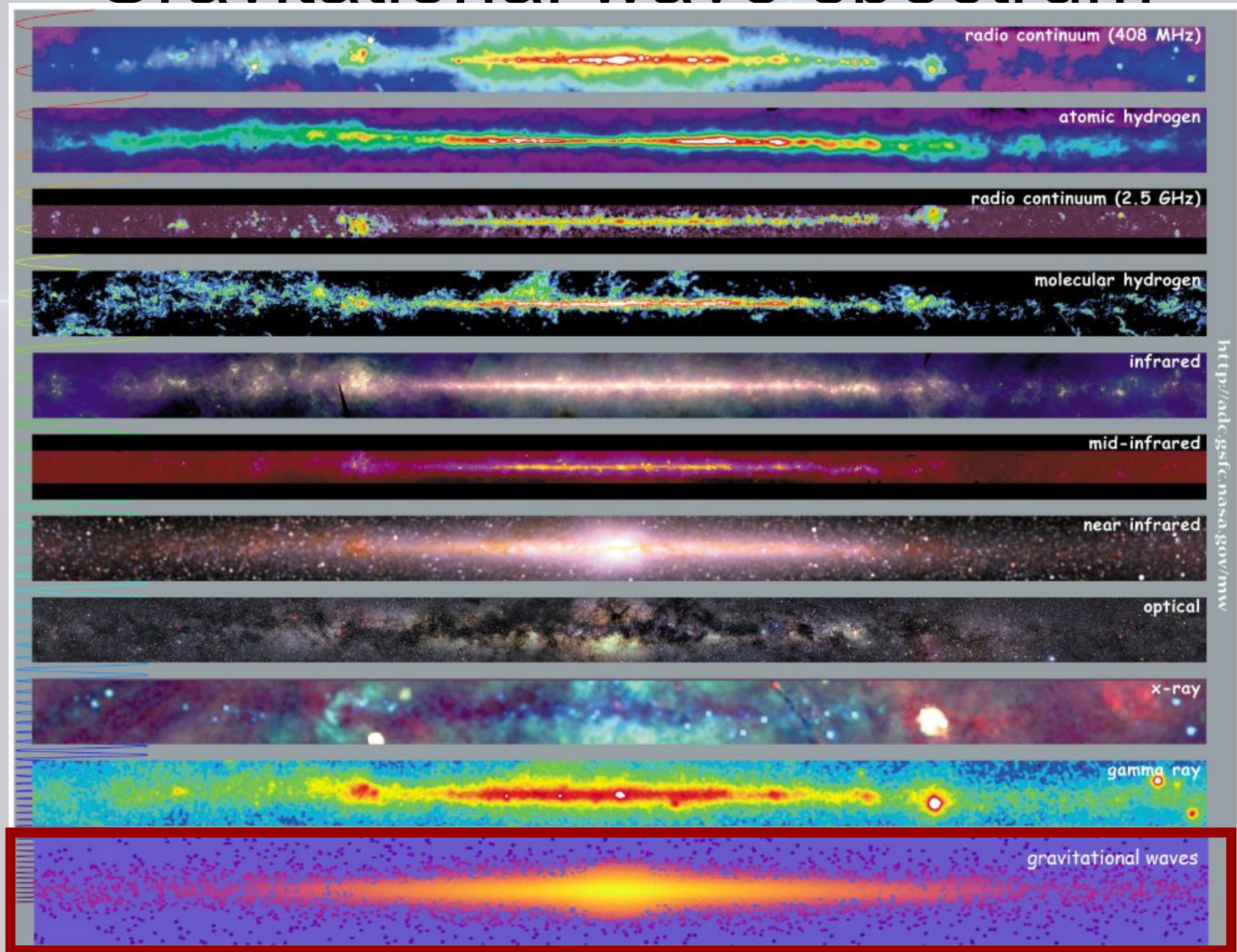
Credit: C. Berry

GO AND CHECK OUT
OTHER SOURCES
AND DETECTORS!!
<http://gwplotter.com/>

Gravitational wave spectrum

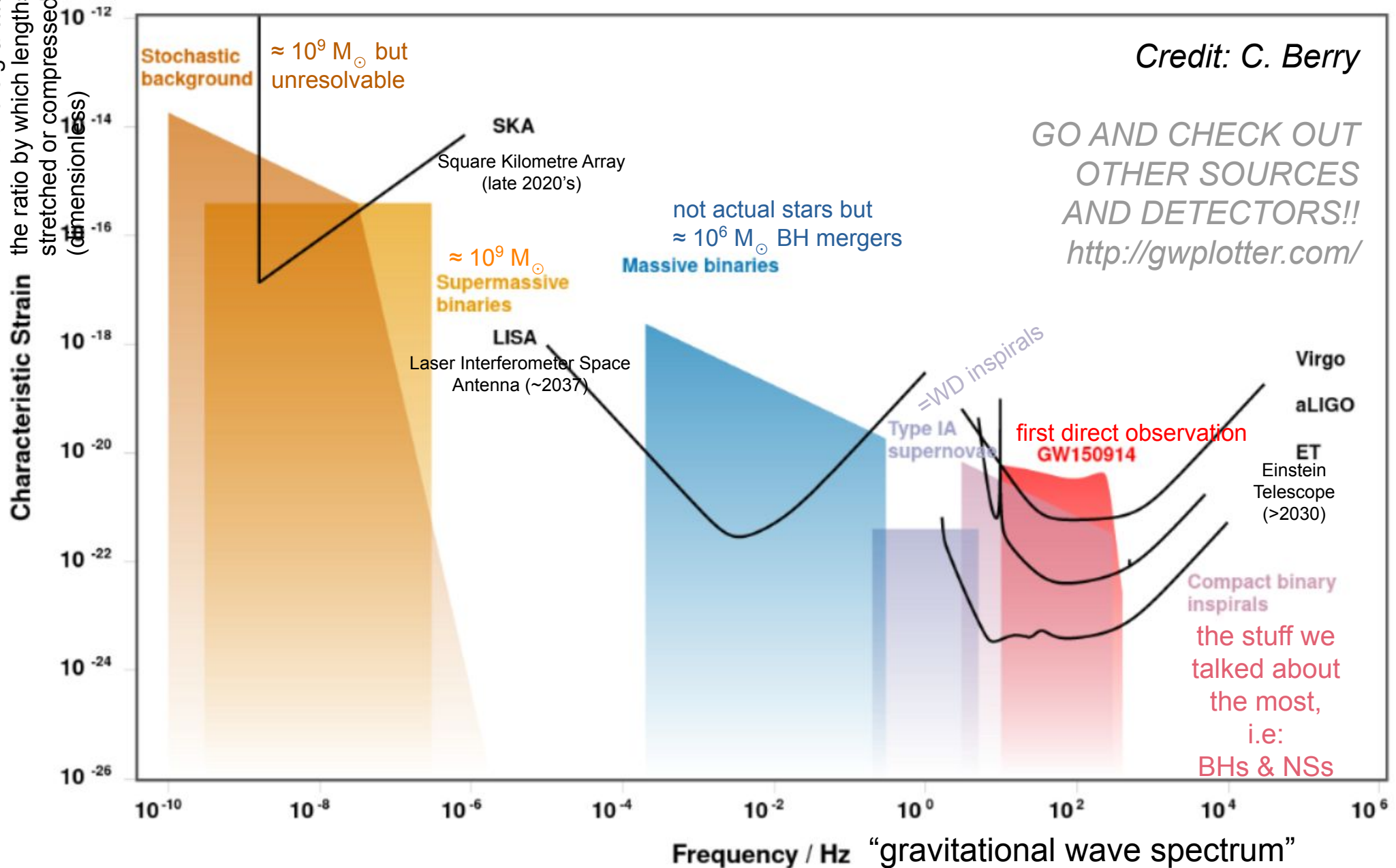


Gravitational wave spectrum



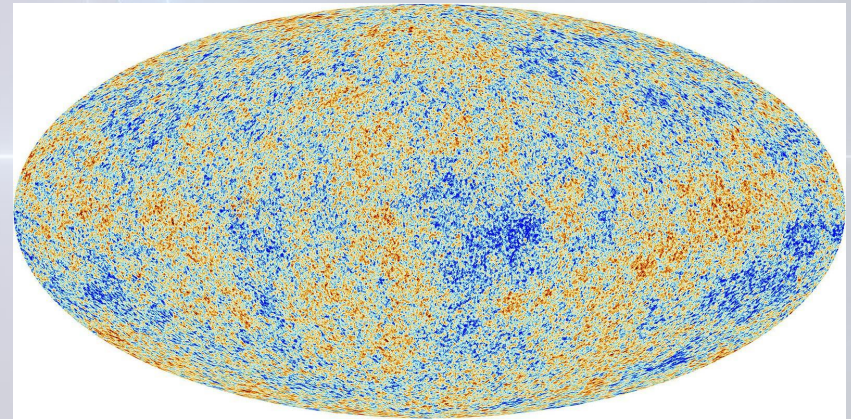
Detector sensibility

a measure of the grav.wave's effect:
the ratio by which lengths are
stretched or compressed
(dimensionless)



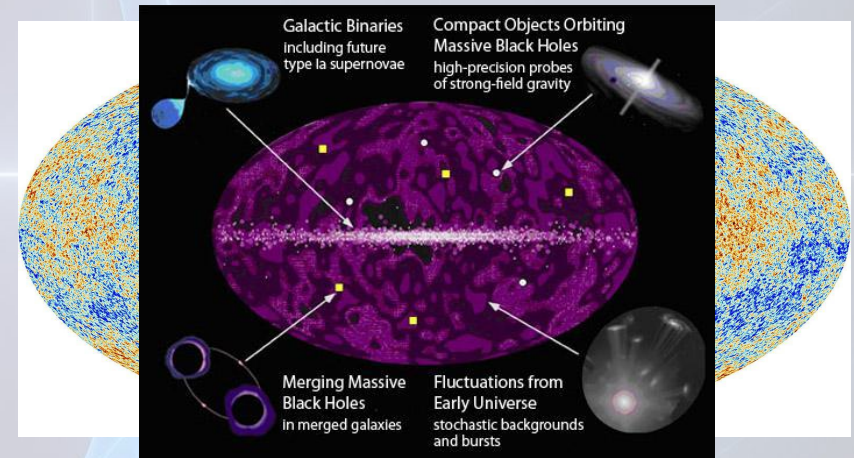
Cosmic grav.wave background

- Heard about the cosmic microwave background?



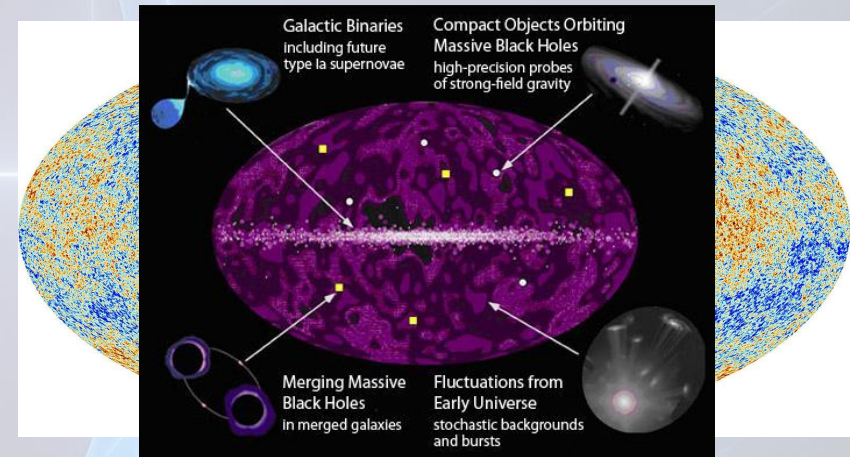
Cosmic grav.wave background

- Heard about the cosmic microwave background?
- GW-background:
 - undetected (yet)



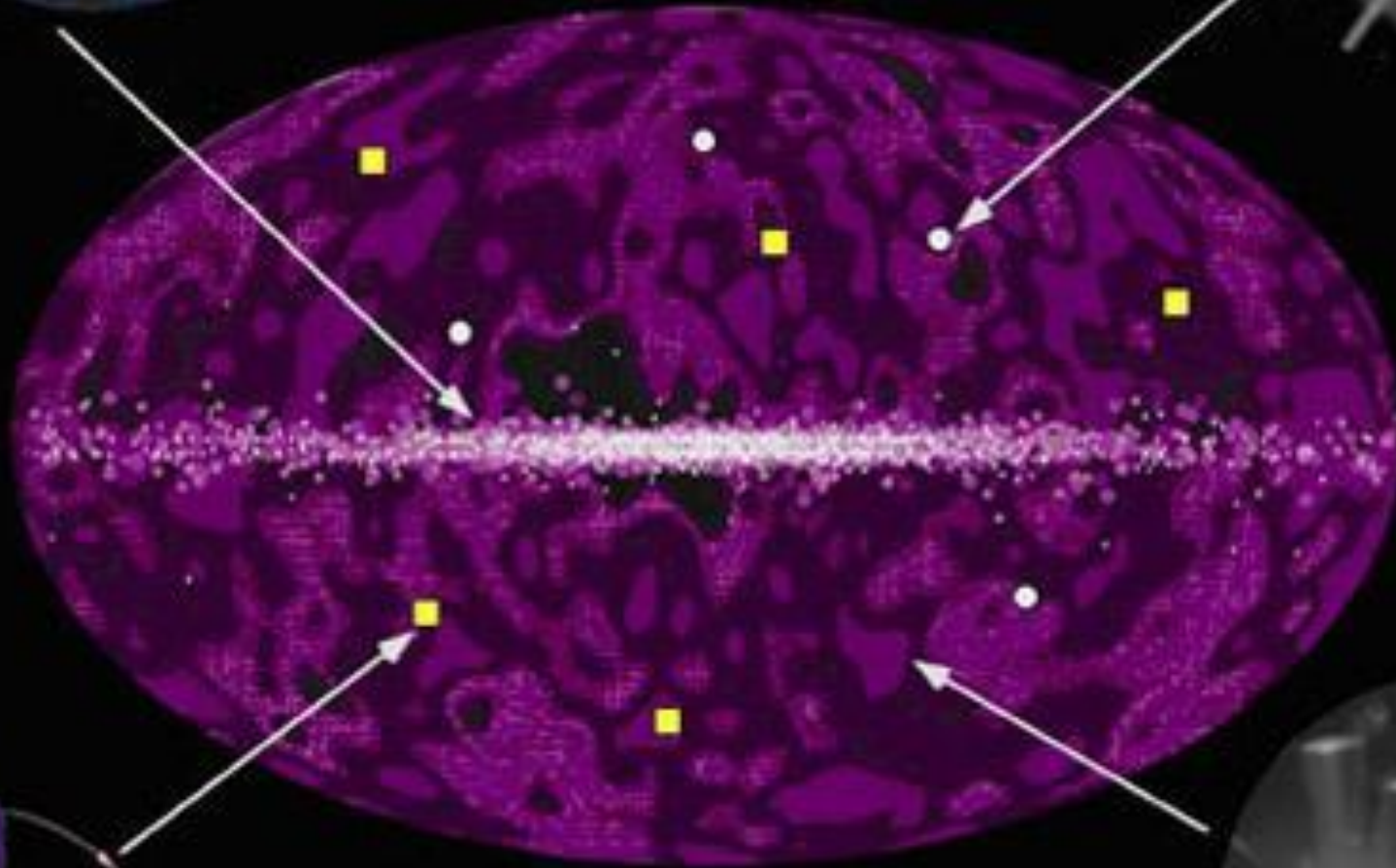
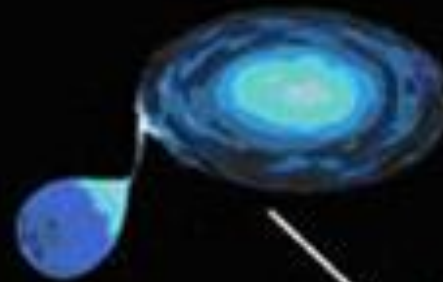
Cosmic grav.wave background

- Heard about the cosmic microwave background?
- GW-background:
 - undetected (yet)
 - cosmological sources
 - processes during e.g. the cosmic inflation (10^{-36} – 10^{-33} sec after the Big Bang)
 - astrophysical sources
 - large number of *unresolvable* BH-BH (or BH-NS, or NS-NS) mergers; additional WD-WD mergers, supernova explosions...



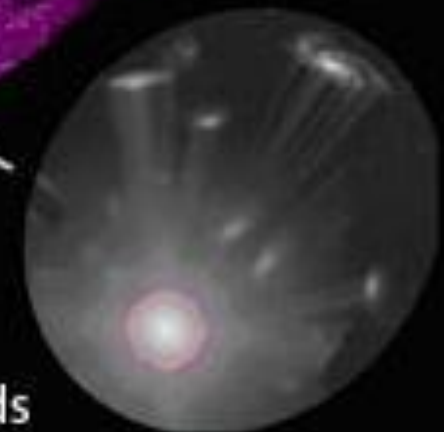
Galactic Binaries
including future
type Ia supernovae

Compact Objects Orbiting
Massive Black Holes
high-precision probes
of strong-field gravity



Merging Massive
Black Holes
in merged galaxies

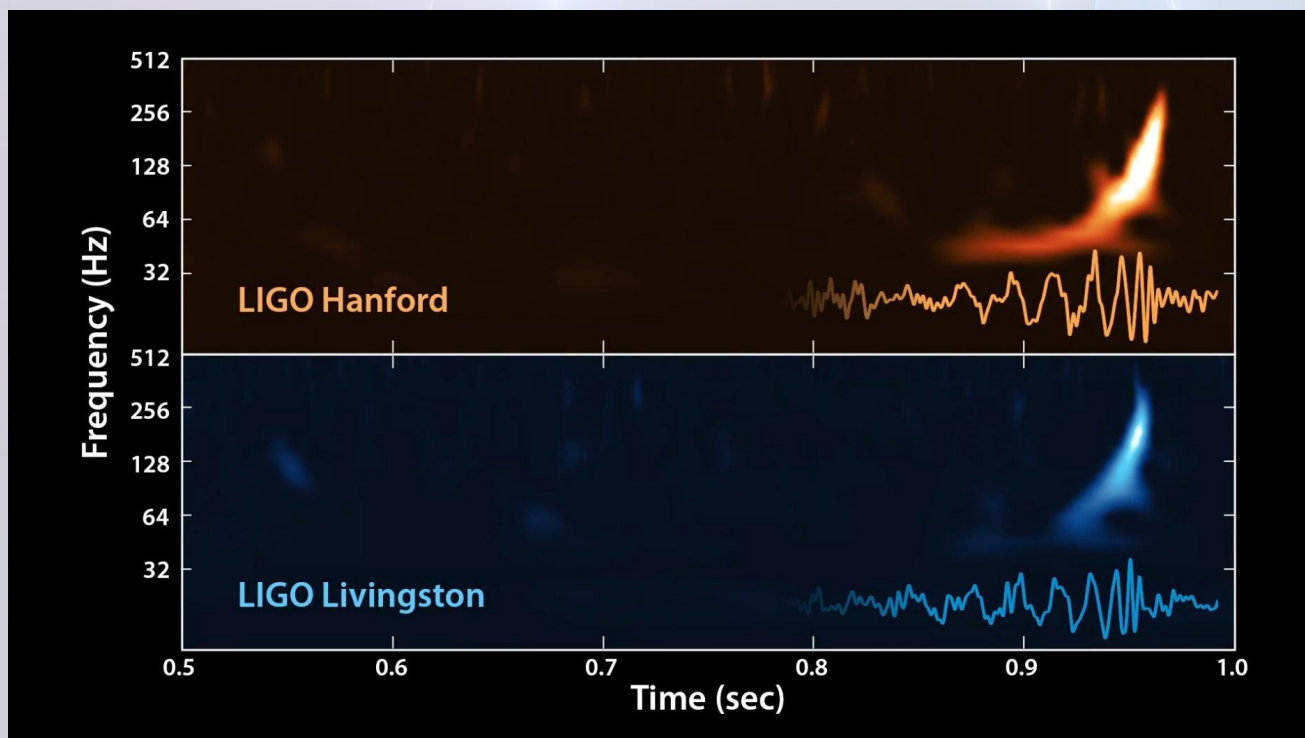
Fluctuations from
Early Universe
stochastic backgrounds
and bursts



Credit: NASA/ESA

The whispering of the Universe

<https://www.youtube.com/watch?v=2PzbYK1x3Vo>



‘GW150914’

$35 M_{\odot}$ & $30 M_{\odot}$

(BH+BH)

=

$64 M_{\odot}$

$3 M_{\odot}$ converted
into GWs!